

1. $\left(\frac{-1+\sqrt{3}i}{2}\right)^{10} + \left(\frac{-1+\sqrt{3}i}{2}\right)^8$ 값을 구하면?

① $\frac{-1+\sqrt{3}i}{2}$

② $\frac{-1-\sqrt{3}i}{2}$

③ 1

④ 0

⑤ -1

해설

$$\omega = \frac{-1+\sqrt{3}i}{2}, 2\omega+1 = \sqrt{3}i$$

양변을 제곱해서 정리하면 $\omega^2 + \omega + 1 = 0$

$$(\omega-1)(\omega^2 + \omega + 1) = 0 \Rightarrow \omega^3 = 1$$

$$(\omega^3)^3 \cdot \omega + (\omega^3)^2 \cdot \omega^2 = \omega + \omega^2 = -1$$

2. $x + \frac{1}{x} = 1$ 일 때, $x^3 + 5x + \frac{2}{x} + \frac{1}{x^3}$ 의 값을 구하면?

- ① $\frac{1}{2}(1 \pm \sqrt{3}i)$ ② $\frac{3}{2}(1 \pm \sqrt{3}i)$ ③ $\frac{5}{2}(2 \pm \sqrt{3}i)$
④ $\frac{7}{2}(3 \pm \sqrt{3}i)$ ⑤ $\frac{9}{2}(4 \pm \sqrt{3}i)$

해설

$$x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0 \therefore x = \frac{1 \pm \sqrt{3}i}{2}$$

$$\begin{aligned}(\text{준식}) &= x^3 + \frac{1}{x^3} + 2\left(x + \frac{1}{x}\right) + 3x \\&= \left(x + \frac{1}{x}\right)^3 - \left(x + \frac{1}{x}\right) + 3x \\&= 3x \\&= \frac{3}{2}(1 \pm \sqrt{3}i)\end{aligned}$$

3. $x = \frac{1 + \sqrt{3}i}{2}$ 일 때, $x^4 - 3x^3 + 3x - 2$ 의 값은?

① $2 + \sqrt{3}i$

② $2 - \sqrt{3}i$

③ $3 + \sqrt{3}i$

④ $-3 + \sqrt{3}i$

⑤ $3 - \sqrt{3}i$

해설

$$x = \frac{1 + \sqrt{3}i}{2}, 2x = 1 + \sqrt{3}i, 2x - 1 = \sqrt{3}i$$

$$4x^2 - 4x + 1 = -3$$

$$4x^2 - 4x + 4 = 0$$

$$x^2 - x + 1 = 0$$

$x^4 - 3x^3 + 3x - 2$ 를 $x^2 - x + 1$ 로 나누면

$$x^4 - 3x^3 + 3x - 2$$

$$= (x^2 - x + 1)(x^2 - 2x - 3) + 2x + 1$$

$$= 0 + 2x + 1$$

$$= 2 \times \frac{1 + \sqrt{3}i}{2} + 1$$

$$= 2 + \sqrt{3}i$$

4. $x = -1 + i$ 일 때, $x^4 + 2x^3 + x^2 - x - 1$ 의 값을 구하면?

① $-1 + i$

② $-i$

③ i

④ -1

⑤ 1

해설

$$x = i - 1 \Rightarrow x + 1 = i$$

$$\text{양변을 제곱해서 정리하면 } x^2 + 2x + 2 = 0$$

$$x^4 + 2x^3 + x^2 - x - 1$$

$$= x^2(x^2 + 2x + 2) - x^2 - x - 1$$

$$= -x^2 - x - 1 \quad (\because x^2 + 2x + 2 = 0)$$

$$= -(-2x - 2) - x - 1$$

$$= x + 1 = i$$

5. 복소수 $z = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$ 에 대하여 $(3z^2 + z)^2 + (z^2 + 3z)^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 2

해설

$$z = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} = \frac{(1 + \sqrt{3}i)^2}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)} \\ = \frac{-1 + \sqrt{3}i}{2}$$

$$z^2 = \frac{-1 - \sqrt{3}i}{2} \\ (3z^2 + z)^2 + (z^2 + 3z)^2 \\ = \left(\frac{-3 - 3\sqrt{3}i - 1 + \sqrt{3}i}{2} \right)^2 \\ + \left(\frac{-1 - \sqrt{3}i - 3 + 3\sqrt{3}i}{2} \right)^2 \\ = (-2 - \sqrt{3}i)^2 + (-2 + \sqrt{3}i)^2 \\ = 4 + 4\sqrt{3}i - 3 + 4 - 4\sqrt{3}i - 3 = 2$$

해설

$$z = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \\ = \frac{(1 + \sqrt{3}i)^2}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)} \\ = \frac{-1 + \sqrt{3}i}{2}$$

에서 양변에 2를 곱하고 -1 을 우변으로 이항하면 $2z + 1 = \sqrt{3}i$

양변을 제곱하면

$$4z^2 + 4z + 1 = -3$$

$$\rightarrow 4z^2 + 4z + 4 = 0$$

$$\rightarrow z^2 + z + 1 = 0$$

$$\rightarrow (z - 1)(z^2 + z + 1) = 0$$

$$\rightarrow z^3 - 1 = 0$$

$$\rightarrow z^3 = 1$$

※방정식에 익숙한 학생들은

$z = \frac{-1 + \sqrt{3}i}{2}$ 에서 바로 $z^2 + z + 1 = 0$ 와 $z^3 = 1$ 을 도출할 수 있을 것이다.

$$(3z^2 + z)^2 + (z^2 + 3z)^2 \\ = 10z^4 + 12z^3 + 10z^2 \\ = (10z^4 + 10z^3 + 10z^2) + 2z^3 \\ = 10z^2(z^2 + z + 1) + 2z^3 \\ = 0 + 2 \\ = 2$$

6. $a = \frac{-1 - \sqrt{3}i}{2}$ 일 때, $a^5 + a^3 - 1$ 의 값을 구하면? (단, $i = \sqrt{-1}$)

① $\frac{1 - \sqrt{3}i}{2}$

② 0

③ 1

④ $\frac{-1 + \sqrt{3}i}{2}$

⑤ $-1 + \sqrt{3}i$

해설

$$a = \frac{-1 - \sqrt{3}i}{2}$$

$2a + 1 = -\sqrt{3}i$ 의 양변을 제곱하면,

$$4a^2 + 4a + 1 = -3 \Rightarrow a^2 + a + 1 = 0$$

양변에 $a - 1$ 를 곱하면

$$(a - 1)(a^2 + a + 1) = 0 \Leftrightarrow a^3 - 1 = 0$$

$$\therefore a^3 = 1$$

$$(\text{준식}) = a^3 a^2 + a^3 - 1$$

$$= a^2$$

$$= -a - 1 (\because a^2 + a + 1 = 0)$$

$$= \frac{-1 + \sqrt{3}i}{2}$$

7. $w = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$ 일 때, $(w + 2w^2)^2 + (2w + w^2)^2$ 의 값을 구하시오.

▶ 답 :

▷ 정답 : 3

해설

$$\begin{aligned}w &= \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} = \frac{-1 + \sqrt{3}i}{2} \\ \therefore w^2 + w + 1 &= 0, \quad w^3 = 1 \\ \therefore (w + 2w^2)^2 + (2w + w^2)^2 & \\ &= (w - 2w - 2)^2 + (2w - w - 1)^2 \\ &= (-w - 2)^2 + (w - 1)^2 \\ &= w^2 + 4w + 4 + w^2 - 2w + 1 \\ &= 2w^2 + 2w + 5 \\ &= 2(w^2 + w + 1) + 3 \\ &= 3\end{aligned}$$

8. $\alpha = \frac{-1 + \sqrt{3}i}{2}$ 일 때, $\alpha^3 + 2\alpha^2 + 2\alpha + 5$ 의 값을 구하면?

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

해설

$$\begin{aligned}\alpha &= \frac{-1 + \sqrt{3}i}{2} \\ 2\alpha &= -1 + \sqrt{3}i \\ 2\alpha + 1 &= \sqrt{3}i \\ \text{양변을 제곱하여 정리하면} \\ \alpha^2 + \alpha + 1 &= 0 \\ \alpha^3 + 2\alpha^2 + 2\alpha + 5 \\ &= \alpha(\alpha^2 + \alpha + 1) + (\alpha^2 + \alpha + 1) + 4 \\ &= 4\end{aligned}$$

해설

$$\begin{aligned}\alpha^2 + \alpha + 1 &= 0 \text{ 을 얻은 후 } \alpha^3 + 2\alpha^2 + 2\alpha + 5 \text{ 를 } \alpha^2 + \alpha + 1 \text{ 로} \\ &\text{나누면} \\ \alpha^3 + 2\alpha^2 + 2\alpha + 5 \\ &= (\alpha^2 + \alpha + 1)(\alpha + 1) + 4 \\ &= 4 \quad (\because \alpha^2 + \alpha + 1 = 0)\end{aligned}$$