

1. $\alpha = \frac{-1 + \sqrt{3}i}{2}$ 일 때, $\alpha^3 + 2\alpha^2 + 2\alpha + 5$ 의 값을 구하면?

- ① 3 ② 4 ③ 5 ④ 6 ⑤ 7

해설

$$\begin{aligned}\alpha &= \frac{-1 + \sqrt{3}i}{2} \\ 2\alpha &= -1 + \sqrt{3}i \\ 2\alpha + 1 &= \sqrt{3}i \\ \text{양변을 제곱하여 정리하면} \\ \alpha^2 + \alpha + 1 &= 0 \\ \alpha^3 + 2\alpha^2 + 2\alpha + 5 \\ &= \alpha(\alpha^2 + \alpha + 1) + (\alpha^2 + \alpha + 1) + 4 \\ &= 4\end{aligned}$$

해설

$$\begin{aligned}\alpha^2 + \alpha + 1 &= 0 \text{ 을 얻은 후 } \alpha^3 + 2\alpha^2 + 2\alpha + 5 \text{ 를 } \alpha^2 + \alpha + 1 \text{ 로} \\ &\text{나누면} \\ \alpha^3 + 2\alpha^2 + 2\alpha + 5 \\ &= (\alpha^2 + \alpha + 1)(\alpha + 1) + 4 \\ &= 4 \quad (\because \alpha^2 + \alpha + 1 = 0)\end{aligned}$$

2. $z = \frac{1+i}{1-i}$ 일 때, $1+z+z^2+\cdots+z^{2008}$ 의 값은?

- ① $-i$ ② -1 ③ 0 ④ i ⑤ 1

해설

$$z = \frac{1+i}{1-i} = i, z^2 = -1, z^3 = -i, z^4 = 1$$

(준식) : $1+z+z^2+z^3+\cdots+z^{2008}$

처음 네 항의 합 :

$$1+i-1-i=0$$

$$1+z+z^2+z^3+\cdots+z^{2008}$$

$$= 0+0+\cdots+0+z^{2008}$$

$$= z^{2008}$$

$$= (z^4)^{502}$$

$$= 1$$

3. 복소수 $z = \frac{2}{1+i}$ 에 대하여 $z^3 - 2z^2 + 2z + 5$ 의 값은?

- ① 1 ② 2 ③ 3 ④ 4 ⑤ 5

해설

$$\begin{aligned} z &= \frac{2}{1+i} = 1-i \\ z^2 &= -2i, \quad z^3 = -2-2i \\ \therefore z^3 - 2z^2 + 2z + 5 \\ &= (-2i-2) - 2(-2i) + 2(1-i) + 5 \\ &= 5 \end{aligned}$$

해설

$$\begin{aligned} z = 1-i &\Rightarrow z-1 = -i \\ &\Rightarrow z^2 - 2z + 1 = -1 \\ &\Rightarrow z^2 - 2z + 2 = 0 \\ z^3 - 2z^2 + 2z + 5 &= z(z^2 - 2z + 2) + 5 = 5 \end{aligned}$$

4. $x + \frac{1}{x} = 1$ 일 때, $x^3 + 5x + \frac{2}{x} + \frac{1}{x^3}$ 의 값을 구하면?

- ① $\frac{1}{2}(1 \pm \sqrt{3}i)$ ② $\frac{3}{2}(1 \pm \sqrt{3}i)$ ③ $\frac{5}{2}(2 \pm \sqrt{3}i)$
④ $\frac{7}{2}(3 \pm \sqrt{3}i)$ ⑤ $\frac{9}{2}(4 \pm \sqrt{3}i)$

해설

$$x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0 \therefore x = \frac{1 \pm \sqrt{3}i}{2}$$

$$\begin{aligned}(\text{준식}) &= x^3 + \frac{1}{x^3} + 2\left(x + \frac{1}{x}\right) + 3x \\&= \left(x + \frac{1}{x}\right)^3 - \left(x + \frac{1}{x}\right) + 3x \\&= 3x \\&= \frac{3}{2}(1 \pm \sqrt{3}i)\end{aligned}$$

5. $a = \frac{-1 - \sqrt{3}i}{2}$ 일 때, $a^5 + a^3 - 1$ 의 값을 구하면? (단, $i = \sqrt{-1}$)

① $\frac{1 - \sqrt{3}i}{2}$

② 0

③ 1

④ $\frac{-1 + \sqrt{3}i}{2}$

⑤ $-1 + \sqrt{3}i$

해설

$$a = \frac{-1 - \sqrt{3}i}{2}$$

$2a + 1 = -\sqrt{3}i$ 의 양변을 제곱하면,

$$4a^2 + 4a + 1 = -3 \Rightarrow a^2 + a + 1 = 0$$

양변에 $a - 1$ 를 곱하면

$$(a - 1)(a^2 + a + 1) = 0 \Leftrightarrow a^3 - 1 = 0$$

$$\therefore a^3 = 1$$

$$(\text{준식}) = a^3 a^2 + a^3 - 1$$

$$= a^2$$

$$= -a - 1 (\because a^2 + a + 1 = 0)$$

$$= \frac{-1 + \sqrt{3}i}{2}$$

6. $x = -1 + i$ 일 때, $x^4 + 2x^3 + x^2 - x - 1$ 의 값을 구하면?

① $-1 + i$

② $-i$

③ i

④ -1

⑤ 1

해설

$$x = i - 1 \Rightarrow x + 1 = i$$

$$\text{양변을 제곱해서 정리하면 } x^2 + 2x + 2 = 0$$

$$x^4 + 2x^3 + x^2 - x - 1$$

$$= x^2(x^2 + 2x + 2) - x^2 - x - 1$$

$$= -x^2 - x - 1 \quad (\because x^2 + 2x + 2 = 0)$$

$$= -(-2x - 2) - x - 1$$

$$= x + 1 = i$$

7. $x = \frac{1 + \sqrt{3}i}{2}$ 일 때, $x^4 - 3x^3 + 3x - 2$ 의 값은?

① $2 + \sqrt{3}i$

② $2 - \sqrt{3}i$

③ $3 + \sqrt{3}i$

④ $-3 + \sqrt{3}i$

⑤ $3 - \sqrt{3}i$

해설

$$x = \frac{1 + \sqrt{3}i}{2}, 2x = 1 + \sqrt{3}i, 2x - 1 = \sqrt{3}i$$

$$4x^2 - 4x + 1 = -3$$

$$4x^2 - 4x + 4 = 0$$

$$x^2 - x + 1 = 0$$

$x^4 - 3x^3 + 3x - 2$ 를 $x^2 - x + 1$ 로 나누면

$$x^4 - 3x^3 + 3x - 2$$

$$= (x^2 - x + 1)(x^2 - 2x - 3) + 2x + 1$$

$$= 0 + 2x + 1$$

$$= 2 \times \frac{1 + \sqrt{3}i}{2} + 1$$

$$= 2 + \sqrt{3}i$$

8. $w = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}$ 일 때, $(w + 2w^2)^2 + (2w + w^2)^2$ 의 값을 구하시오.

▶ 답 :

▷ 정답 : 3

해설

$$\begin{aligned}w &= \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} = \frac{-1 + \sqrt{3}i}{2} \\ \therefore w^2 + w + 1 &= 0, \quad w^3 = 1 \\ \therefore (w + 2w^2)^2 + (2w + w^2)^2 & \\ &= (w - 2w - 2)^2 + (2w - w - 1)^2 \\ &= (-w - 2)^2 + (w - 1)^2 \\ &= w^2 + 4w + 4 + w^2 - 2w + 1 \\ &= 2w^2 + 2w + 5 \\ &= 2(w^2 + w + 1) + 3 \\ &= 3\end{aligned}$$