1. $\alpha = \dfrac{-1+\sqrt{3}i}{2}$ 일 때, $\alpha^3+2\alpha^2+2\alpha+5$ 의 값을 구하면?

① 3 ② 4 ③ 5 ④ 6 ⑤ 7

해설

$\alpha = \dfrac{-1+\sqrt{3}i}{2}$

$2\alpha = -1+\sqrt{3}i$

$2\alpha+1 = \sqrt{3}i$

양변을 제곱하여 정리하면

$\alpha^2+\alpha+1=0$

$\alpha^3+2\alpha^2+2\alpha+5$

$=\alpha(\alpha^2+\alpha+1)+(\alpha^2+\alpha+1)+4$

$=4$

해설

$\alpha^2+\alpha+1=0$ 을 얻은 후 $\alpha^3+2\alpha^2+2\alpha+5$ 를 $\alpha^2+\alpha+1$ 로 나누면

$\alpha^3+2\alpha^2+2\alpha+5$

$=(\alpha^2+\alpha+1)(\alpha+1)+4$

$=4$ ($\because$ $\alpha^2+\alpha+1=0$)

2. $z = \dfrac{1+i}{1-i}$ 일 때, $1 + z + z^2 + \cdots + z^{2008}$ 의 값은?

① $-i$ ② -1 ③ 0 ④ i ⑤ 1

해설

$z = \dfrac{1+i}{1-i} = i, z^2 = -1,\ z^3 = -i,\ z^4 = 1$

(준식) : $1 + z + z^2 + z^3 + \cdots + z^{2008}$

처음 네 항의 합 :

$1 + i - 1 - i = 0$

$1 + z + z^2 + z^3 + \cdots + z^{2008}$

$= 0 + 0 + \cdots + 0 + z^{2008}$

$= z^{2008}$

$= (z^4)^{502}$

$= 1$

3. 복소수 $z = \dfrac{2}{1+i}$ 에 대하여 $z^3 - 2z^2 + 2z + 5$ 의 값은?

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

해설

$$z = \frac{2}{1+i} = 1 - i$$

$$z^2 = -2i,\ z^3 = -2 - 2i$$

$$\therefore\ z^3 - 2z^2 + 2z + 5$$

$$= (-2i - 2) - 2(-2i) + 2(1 - i) + 5$$

$$= 5$$

해설

$$z = 1 - i \quad \Rightarrow \quad z - 1 = -i$$

$$\Rightarrow \quad z^2 - 2z + 1 = -1$$

$$\Rightarrow \quad z^2 - 2z + 2 = 0$$

$$z^3 - 2z^2 + 2z + 5 = z(z^2 - 2z + 2) + 5 = 5$$

4. $x+\frac{1}{x}=1$ 일 때, $x^3+5x+\frac{2}{x}+\frac{1}{x^3}$ 의 값을 구하면?

① $\frac{1}{2}(1\pm\sqrt{3}i)$ ② $\frac{3}{2}(1\pm\sqrt{3}i)$ ③ $\frac{5}{2}(2\pm\sqrt{3}i)$

④ $\frac{7}{2}(3\pm\sqrt{3}i)$ ⑤ $\frac{9}{2}(4\pm\sqrt{3}i)$

해설

$$x+\frac{1}{x}=1 \Rightarrow x^2-x+1=0 \therefore x=\frac{1\pm\sqrt{3}i}{2}$$

$$\begin{aligned}(\text{준식}) &= x^3+\frac{1}{x^3}+2\left(x+\frac{1}{x}\right)+3x \\ &= \left(x+\frac{1}{x}\right)^3-\left(x+\frac{1}{x}\right)+3x \\ &= 3x \\ &= \frac{3}{2}(1\pm\sqrt{3}i)\end{aligned}$$

5. $a = \dfrac{-1 - \sqrt{3}i}{2}$ 일 때, $a^5 + a^3 - 1$ 의 값을 구하면? (단, $i = \sqrt{-1}$)

① $\dfrac{1 - \sqrt{3}i}{2}$ ② 0 ③ 1

④ $\dfrac{-1 + \sqrt{3}i}{2}$ ⑤ $-1 + \sqrt{3}i$

해설

$a = \dfrac{-1 - \sqrt{3}i}{2}$

$2a + 1 = -\sqrt{3}i$ 의 양변을 제곱하면,

$4a^2 + 4a + 1 = -3 \quad \Rightarrow \quad a^2 + a + 1 = 0$

양변에 $a - 1$ 를 곱하면

$(a - 1)(a^2 + a + 1) = 0 \Leftrightarrow a^3 - 1 = 0$

$\therefore \ a^3 = 1$

$(\text{준식}) = a^3a^2 + a^3 - 1$

$= a^2$

$= -a - 1 (\because a^2 + a + 1 = 0)$

$= \dfrac{-1 + \sqrt{3}i}{2}$

6. $x = -1 + i$ 일 때, $x^4 + 2x^3 + x^2 - x - 1$ 의 값을 구하면?

① $-1 + i$　② $-i$　③ i

④ -1　⑤ 1

해설

$x = i - 1 \Rightarrow x + 1 = i$

양변을 제곱해서 정리하면 $x^2 + 2x + 2 = 0$

$x^4 + 2x^3 + x^2 - x - 1$

$= x^2(x^2 + 2x + 2) - x^2 - x - 1$

$= -x^2 - x - 1 \ (\because \ x^2 + 2x + 2 = 0)$

$= -(-2x - 2) - x - 1$

$= x + 1 = i$

7. $x = \dfrac{1+\sqrt{3}i}{2}$ 일 때, $x^4 - 3x^3 + 3x - 2$ 의 값은?

① $2+\sqrt{3}i$ ② $2-\sqrt{3}i$ ③ $3+\sqrt{3}i$

④ $-3+\sqrt{3}i$ ⑤ $3-\sqrt{3}i$

해설

$x = \dfrac{1+\sqrt{3}i}{2}$, $2x = 1+\sqrt{3}i$, $2x-1 = \sqrt{3}i$

$4x^2 - 4x + 1 = -3$

$4x^2 - 4x + 4 = 0$

$x^2 - x + 1 = 0$

$x^4 - 3x^3 + 3x - 2$ 를 $x^2 - x + 1$ 로 나누면

$x^4 - 3x^3 + 3x - 2$

$= (x^2 - x + 1)(x^2 - 2x - 3) + 2x + 1$

$= 0 + 2x + 1$

$= 2 \times \dfrac{1+\sqrt{3}i}{2} + 1$

$= 2+\sqrt{3}i$

8. $w = \dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}$ 일 때, $(w+2w^2)^2+(2w+w^2)^2$ 의 값을 구하시오.

▶ 답 :

▷ 정답 : 3

해설

$$w = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{-1+\sqrt{3}i}{2}$$

$\therefore\ w^2+w+1=0,\quad w^3=1$

$\therefore\ (w+2w^2)^2+(2w+w^2)^2$

$= (w-2w-2)^2+(2w-w-1)^2$

$= (-w-2)^2+(w-1)^2$

$= w^2+4w+4+w^2-2w+1$

$= 2w^2+2w+5$

$= 2(w^2+w+1)+3$

$= 3$