1. 복소수 z의 켤레복소수 $\bar{z}$라 할 때 $(1+2i)z+3(2-\bar{z})=0$을 만족하는 복소수 z를 구하면?

① $z=2-3i$ ② $z=4-3i$ ③ $z=6-3i$
④ $z=2+3i$ ⑤ $z=4+3i$

해설

$z=a+bi, \bar{z}=a-bi$라 하면

(준식) $=(1+2i)(a+bi)+3(2-a+bi)$
$=(6-2a-2b)+(2a+4b)i$

$\therefore 6-2a-2b=0, 2a+4b=0$

$\therefore a=6, b=-3$

$\therefore z=6-3i$

2. $a=1+i$, $b=1-i$ 일 때, $\left(\frac{1}{a}\right)^2+\frac{1}{ab}+\left(\frac{1}{b}\right)^2$ 의 값을 구하면?

① $-\frac{1}{2}$ ② $-\frac{1}{3}$ ③ $\frac{1}{3}$ ④ $\frac{1}{2}$ ⑤ $\frac{1}{4}$

해설

$a^2=(1+i)^2=2i,\ b^2=(1-i)^2=-2i$,
$ab=(1+i)(1-i)=2$

$$\left(\frac{1}{a}\right)^2+\frac{1}{ab}+\left(\frac{1}{b}\right)^2=\frac{b^2+ab+a^2}{a^2b^2}$$
$$=\frac{-2i+2+2i}{4}$$
$$=\frac{1}{2}$$

3. $a<0,\ b<0$ 일 때 다음 중 성립하지 <u>않는</u> 것은?

① $\sqrt{a}\sqrt{b}=\sqrt{ab}$

② $\sqrt{a^3b}=-a\sqrt{ab}$

③ $\dfrac{\sqrt{b}}{\sqrt{a}}=\sqrt{\dfrac{b}{a}}$

④ $\sqrt{\dfrac{b^2}{a}}=\dfrac{b\sqrt{a}}{a}$

⑤ $\sqrt{a^2b}=-a\sqrt{b}$

해설

$a<0,\ b<0$ 이므로,

① $\sqrt{a}\sqrt{b}=\sqrt{-a}i\sqrt{-b}i$

$=\sqrt{-a}\sqrt{-b}i^2$

$=-\sqrt{-a}\sqrt{-b}$

$=-\sqrt{ab}$

$(\because -a>0,\ -b>0)$

따라서, $\sqrt{a}\sqrt{b}=-\sqrt{ab}$

② $\sqrt{a^3b}=\sqrt{a^2\cdot(ab)}$

$=\sqrt{a^2}\sqrt{ab}$

$=|a|\sqrt{ab}$

$=-a\sqrt{ab}$

③ $\dfrac{\sqrt{b}}{\sqrt{a}}=\sqrt{\dfrac{b}{a}}$

④ $\sqrt{\dfrac{b^2}{a}}=\dfrac{|b|}{\sqrt{a}}=-\dfrac{b}{\sqrt{a}}=-\dfrac{b\sqrt{a}}{|a|}=\dfrac{b\sqrt{a}}{a}$

⑤ $\sqrt{a^2b}=-a\sqrt{b}$

4. $f(x)=\left(\frac{1-x}{1+x}\right)^{98}$ 일 때, $f\left(\frac{1-i}{1+i}\right)+f\left(\frac{1+i}{1-i}\right)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : -2

해설

$\frac{1-i}{1+i}=-i$, $\frac{1+i}{1-i}=i$ 이므로

$f(\frac{1-i}{1+i})+f(\frac{1+i}{1-i})$

$=f(-i)+f(i)$

$=\left(\frac{1+i}{1-i}\right)^{98}+\left(\frac{1-i}{1+i}\right)^{98}$

$=i^{98}+(-i)^{98}$

$=i^2+i^2$

$=-2$

5. $\left(\frac{-1+\sqrt{3}i}{2}\right)^{10}+\left(\frac{-1+\sqrt{3}i}{2}\right)^{8}$ 값을 구하면?

① $\frac{-1+\sqrt{3}i}{2}$ ② $\frac{-1-\sqrt{3}i}{2}$ ③ 1

④ 0 ⑤ -1

해설

$\omega=\frac{-1+\sqrt{3}i}{2}$, $2\omega+1=\sqrt{3}i$

양변을 제곱해서 정리하면 $\omega^2+\omega+1=0$

$(\omega-1)(\omega^2+\omega+1)=0 \Rightarrow \omega^3=1$

$(\omega^3)^3\cdot\omega+(\omega^3)^2\cdot\omega^2=\omega+\omega^2=-1$