

1. $x = \frac{a}{b}$, $a \neq b$, $b \neq 0$ 일 때, $\frac{a+b}{a-b}$ 는?

- ① $\frac{x}{x+1}$ ② $\frac{x+1}{x-1}$ ③ 1 ④ $x - \frac{1}{x}$ ⑤ $x + \frac{1}{x}$

해설

$$a = bx \text{ 이므로 } \frac{a+b}{a-b} = \frac{bx+b}{bx-b} = \frac{x+1}{x-1}$$

$$\text{또는 } \frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{x+1}{x-1}$$

2. x 에 대한 항등식 $\frac{6-2x^2}{x^3-x^2-x+1} = \frac{A}{1+x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ 를 만족
시키는 상수 A, B, C 에 대하여 $A^2 + B^2 + C^2$ 의 값은?

- ① 14 ② 13 ③ 12 ④ 11 ⑤ 10

해설

(우변)

$$= \frac{A(x-1)^2 + B(1+x)(1-x) + C(1+x)}{(1+x)(1-x)^2}$$

$$= \frac{(A-B)x^2 + (-2A+C)x + (A+B+C)}{x^3 - x^2 - x + 1}$$

주어진 식의 분자를 비교하면

$$-2x^2 + 6 = (A-B)x^2 + (-2A+C)x + (A+B+C)$$

$$\therefore \begin{cases} A-B = -2 \\ -2A+C = 0 \\ A+B+C = 6 \end{cases}$$

이것을 풀면 $A = 1, B = 3, C = 2$

$$\therefore A^2 + B^2 + C^2 = 1 + 9 + 4 = 14$$

3. 등식 $\frac{3x}{x^3+1} = \frac{a}{x+1} + \frac{bx+c}{x^2-x+1}$ 가 x 에 관한 항등식일 때, $a+b+c$ 의 값은?

① -2 ② -6 ③ 1 ④ 2 ⑤ $\frac{7}{4}$

해설

$$\begin{aligned}\frac{3x}{x^3+1} &= \frac{a}{x+1} + \frac{bx+c}{x^2-x+1} \\ &= \frac{a(x^2-x+1) + (x+1)(bx+c)}{x^3+1} \\ &= \frac{ax^2 - ax + a + bx^2 + bx + cx + c}{x^3+1} \\ &= \frac{(a+b)x^2 + (b-a+c)x + a+c}{x^3+1}\end{aligned}$$

$a+b=0$, $b-a+c=3$, $a+c=0$ 을 연립하여 풀면

$a=-1$, $b=1$, $c=1$

$\therefore a+b+c=1$

4. 등식 $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ 이 x 에 대한 항등식이 될 때, $A-B$ 의 값을 구하면? (단, A, B 는 상수)

① 0

② 1

③ 2

④ 3

⑤ 4

해설

주어진 식의 우변을 정리하면

$$\frac{A(x+1) + Bx}{x(x+1)} = \frac{(A+B)x + A}{x(x+1)}$$

따라서 $\frac{(A+B)x + A}{x(x+1)} = \frac{1}{x(x+1)}$ 이므로

$$A+B=0, A=1$$

$$\therefore B=-1$$

$$\therefore A-B=1-(-1)=2$$

5. 등식 $\frac{a}{x} - \frac{b}{x+1} = \frac{1}{x(x+1)}$ 이 x 에 대한 항등식일 때, 상수 a, b 의 합을 구하면?

① 0 ② 1 ③ 2 ④ 3 ⑤ 4

해설

양변에 $x(x+1)$ 을 곱하면
 $a(x+1) - bx = 1, ax + a - bx - 1 = 0$
 $x(a-b) + (a-1) = 0$
이 등식이 x 에 대한 항등식이므로
 $a = b, a = 1, b = 1$
 $\therefore a + b = 2$

6. 다음 식의 분모를 0으로 만들지 않는 모든 실수 x 에 대하여
- $$\frac{1}{(x-1)(x-2)\cdots(x-10)} = \frac{a_1}{x-1} + \frac{a_2}{x-2} + \cdots + \frac{a_{10}}{x-10}$$
- 이 성립할 때, $a_1 + a_2 + \cdots + a_{10}$ 의 값은?

- ① 0 ② -1 ③ 1 ④ -10 ⑤ 10

해설

$$\begin{aligned} & \frac{1}{(x-1)(x-2)\cdots(x-10)} \\ &= \frac{a_1}{x-1} + \frac{a_2}{x-2} + \cdots + \frac{a_{10}}{x-10} \text{의 양변에} \\ & (x-1)(x-2)\cdots(x-10) \text{을 곱하면} \\ & 1 = a_1(x-2)(x-3)\cdots(x-10) \\ & \quad + a_2(x-1)(x-3)\cdots(x-10) \\ & \quad + \cdots + a_{10}(x-1)(x-2)\cdots(x-9) \\ & 1 = (a_1 + a_2 + \cdots + a_{10})x^9 + \cdots \\ & \text{이 식은 } x \text{에 대한 항등식이므로} \\ & a_1 + a_2 + \cdots + a_{10} = 0 \end{aligned}$$

7. 분수식 $\frac{x}{x+1} + \frac{2x-1}{x-1} - \frac{3x^2+4x+2}{x^2+x}$ 를 간단히 하면?

① $-\frac{x-2}{x(x-1)}$

② $\frac{x+2}{x(x+1)}$

③ $\frac{x-2}{x(x+1)}$

④ $\frac{x+2}{x(x-1)}$

⑤ $\frac{x-2}{x(x-1)}$

해설

$$\begin{aligned} \frac{x}{x+1} &= 1 - \frac{1}{x+1}, \\ \frac{2x-1}{x-1} &= 2 + \frac{1}{x-1}, \\ \frac{3x^2+4x+2}{x^2+x} &= 3 + \frac{x+2}{x^2+x} \quad \text{이므로} \\ (\text{준식}) &= \left(1 - \frac{1}{x+1}\right) + \left(2 + \frac{1}{x-1}\right) \\ &\quad - \left(3 + \frac{x+2}{x^2+x}\right) \\ &= -\frac{1}{x+1} + \frac{1}{x-1} - \frac{x+2}{x(x+1)} \\ &= \frac{-x(x-1) + x(x+1) - (x-1)(x+2)}{x(x+1)(x-1)} \\ &= \frac{-x^2 + x + x^2 + x - x^2 - x + 2}{x(x+1)(x-1)} \\ &= \frac{-x^2 + x + 2}{x(x-1)(x+1)} \\ &= \frac{-(x^2 - x - 2)}{x(x-1)(x+1)} \\ &= \frac{-(x-2)(x+1)}{x(x-1)(x+1)} \\ &= -\frac{x-2}{x(x-1)} \end{aligned}$$

8. $\frac{x+2}{x+1} - \frac{x+3}{x+2} - \frac{x+4}{x+3} + \frac{x+5}{x+4}$ 를 간단히 하면?

- ① $\frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}$
② $\frac{2}{(x+1)(x+2)(x+3)(x+4)}$
③ $\frac{2x}{(x+1)(x+2)(x+3)(x+4)}$
④ $\frac{2(x-1)}{(x+1)(x+2)(x+3)(x+4)}$
⑤ $\frac{2(x-2)}{(x+1)(x+2)(x+3)(x+4)}$

해설

$$\begin{aligned}(\text{준 식}) &= \left(1 + \frac{1}{x+1}\right) - \left(1 + \frac{1}{x+2}\right) \\&\quad - \left(1 + \frac{1}{x+3}\right) + \left(1 + \frac{1}{x+4}\right) \\&= \frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{x+3} + \frac{1}{x+4} \\&= \frac{2x+5}{(x+1)(x+4)} - \frac{2x+5}{(x+2)(x+3)} \\&= \frac{(2x+5)(x^2+5x+6-x^2-5x-4)}{(x+1)(x+2)(x+3)(x+4)} \\&= \frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}\end{aligned}$$

9. 다음 식의 최댓값을 구하면?

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \dots + \frac{1}{(x+9)(x+10)}$$

- ① $\frac{3}{5}$ ② $\frac{2}{5}$ ③ $\frac{1}{5}$ ④ $-\frac{1}{5}$ ⑤ $-\frac{2}{5}$

해설

$$\begin{aligned} \frac{1}{x(x+1)} &= \frac{1}{x} - \frac{1}{x+1}, \\ \frac{1}{(x+1)(x+2)} &= \frac{1}{x+1} - \frac{1}{x+2} \dots \frac{1}{(x+9)(x+10)} \\ &= \frac{1}{x+9} - \frac{1}{x+10} \\ \therefore (\text{준식}) \quad &= \frac{1}{x} - \frac{1}{x+10} = \frac{x+10-x}{x(x+10)} \\ &= \frac{10}{x(x+10)} = \frac{10}{(x+5)^2-25} \\ \therefore \text{최댓값은 } x &= -5 \text{ 일 때 } \frac{10}{-25} = -\frac{2}{5} \end{aligned}$$

10. 부분분수를 이용하여 다음을 만족시키는 양수 x 를 구하여라.

$$\frac{1}{x(x+2)} + \frac{1}{(x+2)(x+4)} + \frac{1}{(x+4)(x+6)} + \frac{1}{(x+6)(x+8)} = \frac{4}{9}$$

▶ 답 :

▷ 정답 : 1

해설

주어진 식을 부분분수로 나타내면

$$\begin{aligned} & \frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+2} \right) + \frac{1}{2} \left(\frac{1}{x+2} - \frac{1}{x+4} \right) \\ & + \frac{1}{2} \left(\frac{1}{x+4} - \frac{1}{x+6} \right) + \frac{1}{2} \left(\frac{1}{x+6} - \frac{1}{x+8} \right) \\ & = \frac{1}{2} \left\{ \left(\frac{1}{x} - \frac{1}{x+2} \right) + \left(\frac{1}{x+2} - \frac{1}{x+4} \right) \right. \\ & \quad \left. + \left(\frac{1}{x+4} - \frac{1}{x+6} \right) + \left(\frac{1}{x+6} - \frac{1}{x+8} \right) \right\} \\ & = \frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+8} \right) = \frac{1}{2} \cdot \frac{8}{x(x+8)} = \frac{4}{x(x+8)} \\ & = \frac{4}{9} \\ & \therefore x(x+8) = 9 \\ & x^2 + 8x - 9 = (x-1)(x+9) = 0 \\ & x > 0 \text{ 이므로 } x = 1 \end{aligned}$$

11. $x = 1$ 일 때,
 $\frac{1}{x(x+1)} + \frac{2}{(x+1)(x+3)} + \frac{3}{(x+3)(x+6)} + \frac{4}{(x+6)(x+10)}$ 의 값
을 구하면?

- ① $\frac{8}{11}$ ② $\frac{10}{11}$ ③ $\frac{12}{11}$ ④ $\frac{8}{9}$ ⑤ $\frac{10}{9}$

해설

이항분리 이용

$$\begin{aligned} & \frac{1}{x(x+1)} + \frac{2}{(x+1)(x+3)} + \frac{3}{(x+3)(x+6)} + \frac{4}{(x+6)(x+10)} \\ &= \frac{1}{x} - \frac{1}{x+1} + \frac{2}{2} \left(\frac{1}{x+1} - \frac{1}{x+3} \right) \\ & \quad + \frac{3}{3} \left(\frac{1}{x+3} - \frac{1}{x+6} \right) + \frac{4}{4} \left(\frac{1}{x+6} - \frac{1}{x+10} \right) \\ &= \frac{1}{x} - \frac{1}{x+10} \\ & x = 1 \text{ 대입하면 } \frac{1}{1} - \frac{1}{1+10} = 1 - \frac{1}{11} = \frac{10}{11} \end{aligned}$$

12. $\frac{1}{x(x+1)} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} - \frac{2x+1}{x(3x+1)}$ 을 간단히 하면 ?

① -2

② -1

③ 0

④ 1

⑤ $\frac{2}{3}$

해설

주어진 식을 이항분리시키면,

$$\begin{aligned}(\text{준식}) &= \left(\frac{1}{x} - \frac{1}{x+1}\right) + \left(\frac{1}{x+1} - \frac{1}{2x+1}\right) \\ &\quad + \left(\frac{1}{2x+1} - \frac{1}{3x+1}\right) - \left(\frac{1}{x} - \frac{1}{3x+1}\right) \\ &= 0\end{aligned}$$

13. $x = 4$ 일 때,
 $\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)}$ 의 값을
구하면?

- ① $\frac{1}{16}$ ② $\frac{1}{8}$ ③ $\frac{1}{4}$ ④ $\frac{1}{2}$ ⑤ 1

해설

$$\begin{aligned} \frac{1}{(x+a)(x+b)} &= \frac{1}{b-a} \left(\frac{1}{x+a} - \frac{1}{x+b} \right) \text{ 이므로} \\ (\text{준식}) &= \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+2} \\ &\quad - \frac{1}{x+3} + \frac{1}{x+3} - \frac{1}{x+4} \\ &= \frac{1}{x} - \frac{1}{x+4} \\ \therefore x = 4 \text{ 대입하면 } &\frac{1}{8} \end{aligned}$$

14. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x+1}$ 일 때, $f(1)g(1) + f(2)g(2) + f(3)g(3) + \cdots + f(49)g(49)$ 의 값을 구하면?

- ① $\frac{48}{49}$ ② $\frac{50}{49}$ ③ $\frac{51}{49}$ ④ $\frac{49}{50}$ ⑤ $\frac{51}{50}$

해설

$$\begin{aligned} f(x)g(x) &= \frac{1}{x} \times \frac{1}{x+1} \\ &= \frac{1}{(x+1)-x} \left(\frac{1}{x} - \frac{1}{x+1} \right) \\ &= \frac{1}{x} - \frac{1}{x+1} \text{ 이므로} \end{aligned}$$

(주어진 식) =

$$\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{49} - \frac{1}{50} \right) = 1 - \frac{1}{50} = \frac{49}{50}$$

15. 다음의 식을 간단히 하면?

$$\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{119}+\sqrt{121}}$$

- ① 5
- ② 10
- ③ 0
- ④ -10
- ⑤ -5

해설

준식을 유리화하면

$$\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\cdots+\frac{\sqrt{121}-\sqrt{119}}{2}$$

$$=-\frac{1}{2}+\frac{\sqrt{121}}{2}=\frac{11-1}{2}=5$$

16. 다음 중 $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42}$ 을 간단히 한 것은?

- ① $\frac{2}{13}$
- ② $\frac{4}{13}$
- ③ $\frac{5}{14}$
- ④ $\frac{23}{30}$
- ⑤ $\frac{31}{42}$

해설

$$\begin{aligned} &\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} \\ &= \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} \\ &= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) \\ &= \frac{1}{2} - \frac{1}{7} = \frac{5}{14} \end{aligned}$$

17. $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{13 \times 14} = \frac{a}{14}$ 에서 a 의 값을 구하여라.

▶ 답 :

▷ 정답 : 13

해설

$$\begin{aligned} \text{준식} &= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \cdots - \frac{1}{14} = 1 - \frac{1}{14} = \frac{13}{14} \\ \therefore a &= 13 \end{aligned}$$

18. 다음은 $\frac{x^2-x-3}{x-1} - \frac{x^2+x-1}{x+1}$ 를 계산하는 과정이다. 다음 중 ㉠, ㉡, ㉢, ㉣을 차례대로 구하고 풀이를 완성하여 그 값을 바르게 구한 것은?

$$\frac{x^2-x-3}{x-1} = (\textcircled{\text{㉠}}) + \frac{(\textcircled{\text{㉡}})}{x-1}$$

$$\frac{x^2+x-1}{x+1} = (\textcircled{\text{㉢}}) + \frac{(\textcircled{\text{㉣}})}{x+1}$$

- ㉠ $-x, +3, x, -1, \frac{2x+4}{x^2-1}$

㉡ $x, -3, x, -1, -\frac{2x+4}{x^2-1}$
- ㉢ $x, 3, x, 1, -\frac{2x+4}{x^2+1}$

㉣ $x, -1, x, -3, -\frac{2x-4}{x^2-1}$
- ㉤ $x, 1, x, 3, -\frac{2x+4}{x^2+1}$

해설

$$\frac{x^2-x-3}{x-1} = \frac{x(x-1)-3}{x-1} = x + \frac{-3}{x-1}$$

$$\frac{x^2+x-1}{x+1} = \frac{x(x+1)-1}{x+1} = x + \frac{-1}{x+1}$$

$$\therefore \textcircled{\text{㉠}} = x, \textcircled{\text{㉡}} = -3, \textcircled{\text{㉢}} = x, \textcircled{\text{㉣}} = -1$$

$$\begin{aligned} \text{(준식)} &= x - \frac{3}{x-1} - \left(x - \frac{1}{x+1}\right) \\ &= \frac{1}{x+1} - \frac{3}{x-1} \\ &= \frac{x-1-3(x+1)}{(x+1)(x-1)} \\ &= -\frac{2x+4}{x^2-1} \end{aligned}$$

19. $\frac{\sqrt{2}-1}{\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}} + \frac{\sqrt{4}-\sqrt{3}}{\sqrt{12}} + \frac{\sqrt{5}-\sqrt{4}}{\sqrt{20}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{30}}$ 의 값은?

① $\frac{6-\sqrt{6}}{6}$

② $\frac{\sqrt{5}-1}{12}$

③ $\frac{10-\sqrt{2}}{20}$

④ $\frac{16-\sqrt{5}}{30}$

⑤ $\frac{\sqrt{30}-1}{2}$

해설

$\sqrt{2} = \sqrt{1} \times \sqrt{2}$, $\sqrt{6} = \sqrt{2} \times \sqrt{3}$, $\dots$, $\sqrt{30} = \sqrt{5} \times \sqrt{6}$ 임을 이용한다.

$$\begin{aligned} & \frac{\sqrt{2}-1}{\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}} + \frac{\sqrt{4}-\sqrt{3}}{\sqrt{12}} + \frac{\sqrt{5}-\sqrt{4}}{\sqrt{20}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{30}} \\ &= \frac{\sqrt{2}-1}{\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3} \times \sqrt{2}} + \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4} \times \sqrt{3}} + \frac{\sqrt{5}-\sqrt{4}}{\sqrt{5} \times \sqrt{4}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6} \times \sqrt{5}} \\ &= \left(1 - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}\right) \\ & \quad + \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}}\right) \\ &= 1 - \frac{1}{\sqrt{6}} = \frac{\sqrt{6}-1}{\sqrt{6}} = \frac{6-\sqrt{6}}{6} \end{aligned}$$