

1. $\frac{x-2}{2x^2-5x+3} + \frac{3x-1}{2x^2+x-6} + \frac{2x^2-5}{x^2+x-2}$ 을 계산하여라.

▶ 답 :

▷ 정답 : 2

해설

(준 식)

$$\begin{aligned} &= \frac{x-2}{(2x-3)(x-1)} + \frac{3x-1}{(2x-3)(x+2)} + \frac{2x^2-5}{x^2+x-2} \\ &= \frac{(x-2)(x+2) + (3x-1)(x-1)}{(2x-3)(x-1)(x+2)} + \frac{2x^2-5}{(x+2)(x-1)} \\ &= \frac{4x^2-4x-3}{(2x-3)(x-1)(x+2)} + \frac{2x^2-5}{(x+2)(x-1)} \\ &= \frac{(2x-3)(2x+1)}{(2x-3)(x+2)(x-1)} + \frac{2x^2-5}{(x+2)(x-1)} \\ &= \frac{2x+1}{(x+2)(x-1)} + \frac{2x^2-5}{(x+2)(x-1)} \\ &= \frac{2x^2+2x-4}{(x+2)(x-1)} = 2 \end{aligned}$$

2. 분수식 $\frac{1}{x^2+x-2} - \frac{x+1}{x^2-4x+3} \div \frac{2x^2+5x+3}{x^2-5x+6}$ 을 간단히 하면 ?

① 1

② -2

③ $\frac{-x^2+2x+7}{(x-1)(x+2)(2x+3)}$

④ $\frac{x^2-2x+7}{(x-1)(x+2)(2x+3)}$

⑤ $\frac{-x^2+2x+7}{(x+1)(x-2)(2x-3)}$

해설

$$\begin{aligned} & \frac{1}{x^2+x-2} - \frac{x+1}{x^2-4x+3} \div \frac{2x^2+5x+3}{x^2-5x+6} \\ &= \frac{1}{(x+2)(x-1)} - \frac{x+1}{(x-3)(x-1)} \\ & \times \frac{(x-2)(x-3)}{(2x+3)(x+1)} \\ &= \frac{2x+3-(x-2)(x+2)}{(x+2)(x-1)(2x+3)} \\ &= \frac{-x^2+2x+7}{(x-1)(x+2)(2x+3)} \end{aligned}$$

3. $\frac{x+1}{x(x-1)} = \frac{a}{x} + \frac{b}{x-1}$ 가 x 에 대한 항등식일 때, 상수 $a^2 + b^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\frac{x+1}{x(x-1)} = \frac{(a+b)x-a}{x(x-1)}$$

따라서, $a+b=1$, $a=-1$

$\therefore a=-1$, $b=2$

$\therefore a^2 + b^2 = (-1)^2 + 2^2 = 5$

4. 분수식 $\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)}$ 을 간단히 하면?

① $\frac{2}{x(x+1)}$

② $\frac{1}{x(x+2)}$

③ $\frac{1}{x(x+1)}$

④ $\frac{2}{x(x+2)}$

⑤ $\frac{3}{x(x+2)}$

해설

$$\begin{aligned}\frac{1}{x(x+1)} &= \frac{1}{(x+1)-x} \left(\frac{1}{x} - \frac{1}{x+1} \right) \\ &= \frac{1}{x} - \frac{1}{x+1} \\ \frac{1}{(x+1)(x+2)} &= \frac{1}{(x+2)-(x+1)} \left(\frac{1}{x+1} - \frac{1}{x+2} \right) \\ &= \frac{1}{x+1} - \frac{1}{x+2} \\ \therefore (\text{준식}) &= \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x+1} - \frac{1}{x+2} \\ &= \frac{1}{x} - \frac{1}{x+2} = \frac{2}{x(x+2)}\end{aligned}$$

5. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{99 \cdot 100}$ 을 간단히 하면?

① $\frac{98}{99}$

② $\frac{100}{99}$

③ $\frac{99}{100}$

④ $\frac{101}{100}$

⑤ $\frac{100}{101}$

해설

이항분리 이용

$$\begin{aligned} & \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100} \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{99} - \frac{1}{100} \\ &= 1 - \frac{1}{100} = \frac{99}{100} \end{aligned}$$

6. 다음 식을 간단히 한 식은?

$$\frac{\frac{1}{1-\frac{1}{1-\frac{1}{a}}}}$$

① $a+1$

② $a+2$

③ $-a+1$

④ $-a+2$

⑤ $a-1$

해설

아래에서부터 계산해 올라가자.

$$\frac{1}{1-\frac{1}{1-\frac{1}{a}}} = \frac{1}{1-\frac{a}{a-1}} = \frac{a-1}{a-1-a} = -a+1$$

7. $\frac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}}$ 를 풀면?

- ① 6 ② 8 ③ $\frac{31}{2}$ ④ 24 ⑤ 512

해설

분자, 분모에 2^3 을 곱하면

$$\frac{2^3(2^1 + 2^0 + 2^{-1})}{2^3(2^{-2} + 2^{-3} + 2^{-4})} = \frac{2^3(2^1 + 2^0 + 2^{-1})}{2^1 + 2^0 + 2^{-1}} \\ = 2^3 = 8$$

해설

$$\frac{2 + 1 + \frac{1}{2}}{\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}} = \frac{\frac{7}{2}}{\frac{1}{4} + \frac{1}{8} + \frac{1}{16}} = \frac{\frac{7}{2}}{\frac{7}{16}} = 8$$

8. $\frac{x^3-x}{x^2-x} + \frac{x^4-1}{x^2-1} - \frac{x^2-2x-3}{x+1} \times \frac{x+2}{x^2-x-6}$ 을 계산하면?

- ① x^2+x+1 ② $\frac{x^2+1}{x-1}$ ③ $\frac{2x}{x^2-1}$
④ x^2-1 ⑤ $\frac{2x-1}{x^2-x}$

해설

$$\begin{aligned} & \frac{x(x+1)(x-1)}{x(x-1)} + \frac{(x^2+1)(x^2-1)}{x^2-1} \\ & - \frac{(x+1)(x-3)}{x+1} \times \frac{x+2}{(x-3)(x+2)} \\ & = x+1+x^2+1-1 = x^2+x+1 \end{aligned}$$

9. $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{9^2}\right)\left(1 - \frac{1}{10^2}\right)$ 을 간단히 하면?

① $\frac{5}{12}$

② $\frac{1}{2}$

③ $\frac{11}{20}$

④ $\frac{2}{3}$

⑤ $\frac{7}{10}$

해설

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{9^2}\right)\left(1 - \frac{1}{10^2}\right)$$

$$= \frac{1 \cdot 3}{2^2} \cdot \frac{2 \cdot 4}{3^2} \cdots \frac{8 \cdot 10}{9^2} \cdot \frac{9 \cdot 11}{10^2}$$

$$= \frac{1 \cdot 11}{2 \cdot 10} = \frac{11}{20}$$

일반적으로

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

10. $\frac{1}{a(a+1)} + \frac{2}{(a+1)(a+3)} + \frac{3}{(a+3)(a+6)}$ 을 간단히 한 것은 ?

- ① $\frac{1}{a} + \frac{6}{a+6}$ ② $\frac{1}{a} + \frac{1}{a+6}$ ③ $\frac{1}{a} - \frac{1}{a+6}$
④ $\frac{1}{a} - \frac{6}{a+6}$ ⑤ $\frac{2}{a} - \frac{1}{a+6}$

해설

(준식)

$$\begin{aligned} &= \frac{1}{a} - \frac{1}{a+1} + \frac{1}{a+1} - \frac{1}{a+3} + \frac{1}{a+3} - \frac{1}{a+6} \\ &= \frac{1}{a} - \frac{1}{a+6} \end{aligned}$$

11. 분수식 $\frac{x^2}{(x-y)(x-z)} + \frac{y^2}{(y-x)(y-z)} + \frac{z^2}{(z-x)(z-y)}$ 를 간단히 하여라.

▶ 답 :

▷ 정답 : 1

해설

$$\frac{x^2(z-y) + y^2(z-x) + z^2(y-x)}{(x-y)(y-z)(z-x)} \dots \textcircled{1}$$

①에서 분자를 x 에 관하여 정리하면

$$\begin{aligned} & x^2(z-y) + y^2(z-x) + z^2(y-x) \\ &= (z-y)x^2 - (z^2-y^2)x + yz^2 - y^2z \\ &= (z-y)x^2 - (z+y)(z-y)x + zy(z-y) \\ &= (z-y)\{x^2 - (z+y)x + zy\} \\ &= (z-y)(x-z)(x-y) = (x-y)(y-z)(z-x) \\ \therefore (\text{준식}) &= \frac{(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)} = 1 \end{aligned}$$

12. 다음 식을 간단히 하면?

$$\frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} - \frac{4}{1+x^4}$$

- ① $\frac{8x^4}{1-x^4}$ ② $\frac{8}{1-x^4}$ ③ $\frac{8x^4}{1-x^8}$
④ $\frac{8}{1-x^8}$ ⑤ $\frac{8x^4}{1+x^8}$

해설

$$\begin{aligned} & \frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} - \frac{4}{1+x^4} \\ &= \frac{2}{1-x^2} + \frac{2}{1+x^2} - \frac{4}{1+x^4} \\ &= \frac{4}{1-x^4} - \frac{4}{1+x^4} = \frac{8x^4}{1-x^8} \end{aligned}$$

13. $x = \frac{a}{b}$, $a \neq b$, $b \neq 0$ 일 때, $\frac{a+b}{a-b}$ 는?

- ① $\frac{x}{x+1}$ ② $\frac{x+1}{x-1}$ ③ 1 ④ $x - \frac{1}{x}$ ⑤ $x + \frac{1}{x}$

해설

$$a = bx \text{ 이므로 } \frac{a+b}{a-b} = \frac{bx+b}{bx-b} = \frac{x+1}{x-1}$$

$$\text{또는 } \frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{x+1}{x-1}$$

14. $\frac{x+3}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}$ 을 만족할 때, $a^2 + b^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\begin{aligned}\frac{x+3}{(x+1)(x+2)} &= \frac{a}{x+1} + \frac{b}{x+2} \\ &= \frac{(a+b)x + 2a + b}{(x+1)(x+2)}\end{aligned}$$

$$a+b=1, 2a+b=3$$

$$\therefore a=2, b=-1$$

$$\therefore a^2 + b^2 = 2^2 + (-1)^2 = 5$$

15. 등식 $\frac{x^2+1}{x^3-6x^2+11x-6} = \frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x-3}$ 이 x 에 대한 항등식 이 되도록 상수 a, b, c 에 대하여 abc 의 값을 구하여라.

▶ 답 :

▷ 정답 : -25

해설

$x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$ 이므로

양변에 $(x-1)(x-2)(x-3)$ 을 곱하면

$x^2 + 1 = a(x-2)(x-3) + b(x-1)(x-3) + c(x-1)(x-2)$

양변에 $x = 1$ 을 대입하면 $2 = 2a$

$\therefore a = 1$

양변에 $x = 2$ 를 대입하면 $5 = -b$

$\therefore b = -5$

양변에 $x = 3$ 을 대입하면 $10 = 2c$

$\therefore c = 5$

$\therefore abc = -25$

16. 등식 $\frac{a}{x} - \frac{b}{x+1} = \frac{1}{x(x+1)}$ 이 x 에 대한 항등식일 때, 상수 a, b 의 합을 구하면?

① 0

② 1

③ 2

④ 3

⑤ 4

해설

양변에 $x(x+1)$ 을 곱하면

$$a(x+1) - bx = 1, ax + a - bx - 1 = 0$$

$$x(a-b) + (a-1) = 0$$

이 등식이 x 에 대한 항등식이므로

$$a = b, a = 1, b = 1$$

$$\therefore a + b = 2$$

17. 분수식 $\frac{x}{x+1} + \frac{2x-1}{x-1} - \frac{3x^2+4x+2}{x^2+x}$ 를 간단히 하면?

- ① $-\frac{x-2}{x(x-1)}$ ② $\frac{x+2}{x(x+1)}$ ③ $\frac{x-2}{x(x+1)}$
 ④ $\frac{x+2}{x(x-1)}$ ⑤ $\frac{x-2}{x(x-1)}$

해설

$$\begin{aligned}
 \frac{x}{x+1} &= 1 - \frac{1}{x+1}, \\
 \frac{2x-1}{x-1} &= 2 + \frac{1}{x-1}, \\
 \frac{3x^2+4x+2}{x^2+x} &= 3 + \frac{x+2}{x^2+x} \quad \text{이므로} \\
 (\text{준식}) &= \left(1 - \frac{1}{x+1}\right) + \left(2 + \frac{1}{x-1}\right) \\
 &\quad - \left(3 + \frac{x+2}{x^2+x}\right) \\
 &= -\frac{1}{x+1} + \frac{1}{x-1} - \frac{x+2}{x(x+1)} \\
 &= \frac{-x(x-1) + x(x+1) - (x-1)(x+2)}{x(x+1)(x-1)} \\
 &= \frac{-x^2 + x + x^2 + x - x^2 - x + 2}{x(x+1)(x-1)} \\
 &= \frac{-x^2 + x + 2}{x(x-1)(x+1)} \\
 &= \frac{-(x^2 - x - 2)}{x(x-1)(x+1)} \\
 &= \frac{-(x-2)(x+1)}{x(x-1)(x+1)} \\
 &= -\frac{x-2}{x(x-1)}
 \end{aligned}$$

18. $\frac{x+2}{x+1} - \frac{x+3}{x+2} - \frac{x+4}{x+3} + \frac{x+5}{x+4}$ 를 간단히 하면?

- ① $\frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}$
② $\frac{2}{(x+1)(x+2)(x+3)(x+4)}$
③ $\frac{2x}{(x+1)(x+2)(x+3)(x+4)}$
④ $\frac{2(x-1)}{(x+1)(x+2)(x+3)(x+4)}$
⑤ $\frac{2(x-2)}{(x+1)(x+2)(x+3)(x+4)}$

해설

$$\begin{aligned}(\text{준 식}) &= \left(1 + \frac{1}{x+1}\right) - \left(1 + \frac{1}{x+2}\right) \\&\quad - \left(1 + \frac{1}{x+3}\right) + \left(1 + \frac{1}{x+4}\right) \\&= \frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{x+3} + \frac{1}{x+4} \\&= \frac{2x+5}{(x+1)(x+4)} - \frac{2x+5}{(x+2)(x+3)} \\&= \frac{(2x+5)(x^2+5x+6-x^2-5x-4)}{(x+1)(x+2)(x+3)(x+4)} \\&= \frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}\end{aligned}$$

19. 부분분수를 이용하여 다음을 만족시키는 양수 x 를 구하여라.

$$\frac{1}{x(x+2)} + \frac{1}{(x+2)(x+4)} + \frac{1}{(x+4)(x+6)} + \frac{1}{(x+6)(x+8)} = \frac{4}{9}$$

▶ 답 :

▷ 정답 : 1

해설

주어진 식을 부분분수로 나타내면

$$\begin{aligned} & \frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+2} \right) + \frac{1}{2} \left(\frac{1}{x+2} - \frac{1}{x+4} \right) \\ & + \frac{1}{2} \left(\frac{1}{x+4} - \frac{1}{x+6} \right) + \frac{1}{2} \left(\frac{1}{x+6} - \frac{1}{x+8} \right) \\ & = \frac{1}{2} \left\{ \left(\frac{1}{x} - \frac{1}{x+2} \right) + \left(\frac{1}{x+2} - \frac{1}{x+4} \right) \right. \\ & \quad \left. + \left(\frac{1}{x+4} - \frac{1}{x+6} \right) + \left(\frac{1}{x+6} - \frac{1}{x+8} \right) \right\} \\ & = \frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+8} \right) = \frac{1}{2} \cdot \frac{8}{x(x+8)} = \frac{4}{x(x+8)} \\ & = \frac{4}{9} \\ & \therefore x(x+8) = 9 \\ & x^2 + 8x - 9 = (x-1)(x+9) = 0 \\ & x > 0 \text{ 이므로 } x = 1 \end{aligned}$$

20. $\frac{1}{x(x+1)} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} - \frac{2x+1}{x(3x+1)}$ 을 간단히 하면 ?

- ① -2 ② -1 ③ 0 ④ 1 ⑤ $\frac{2}{3}$

해설

주어진 식을 이항분리시키면,

$$\begin{aligned}(\text{준식}) &= \left(\frac{1}{x} - \frac{1}{x+1}\right) + \left(\frac{1}{x+1} - \frac{1}{2x+1}\right) \\ &\quad + \left(\frac{1}{2x+1} - \frac{1}{3x+1}\right) - \left(\frac{1}{x} - \frac{1}{3x+1}\right) \\ &= 0\end{aligned}$$

21. 분수식

$$\frac{1}{x(x+1)} + \frac{2}{(x+1)(x+3)} + \frac{3}{(x+3)(x+6)} \\ + \frac{4}{(x+6)(x+10)} + \frac{5}{(x+10)(x+15)} \\ + \frac{6}{(x+15)(x+21)} + \frac{7}{(x+21)(x+28)} \text{을 간단히 하면?}$$

- ① $\frac{28}{x^2+28x}$
- ② $\frac{7}{x^2+28x}$
- ③ $\frac{7}{x^2+21x}$
- ④ $\frac{6}{x^2+21x}$
- ⑤ $\frac{1}{x^2+21x}$

해설

$$\begin{aligned} (\text{준식}) &= \left(\frac{1}{x} - \frac{1}{x+1}\right) + \left(\frac{1}{x+1} - \frac{1}{x+3}\right) + \cdots \\ &\quad + \left(\frac{1}{x+21} - \frac{1}{x+28}\right) \\ &= \frac{1}{x} - \frac{1}{x+28} = \frac{28}{x^2+28x} \end{aligned}$$

22. $\frac{1}{2 \cdot 4} + \frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \cdots + \frac{1}{18 \cdot 20}$ 을 계산한 값은?

① 0

② $\frac{9}{20}$

③ 40

④ $\frac{40}{9}$

⑤ $\frac{9}{40}$

해설

$$\begin{aligned} & \frac{1}{2} \left\{ \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{6} \right) + \cdots + \left(\frac{1}{18} - \frac{1}{20} \right) \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{2} + \left(-\frac{1}{4} + \frac{1}{4} \right) + \left(-\frac{1}{6} + \frac{1}{6} \right) + \cdots - \frac{1}{20} \right\} \\ &= \frac{1}{2} \left(\frac{1}{2} - \frac{1}{20} \right) = \frac{1}{2} \cdot \frac{9}{20} = \frac{9}{40} \end{aligned}$$

23. 다음의 식을 간단히 하면?

$$\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{119}+\sqrt{121}}$$

- ① 5
- ② 10
- ③ 0
- ④ -10
- ⑤ -5

해설

준식을 유리화하면

$$\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\cdots+\frac{\sqrt{121}-\sqrt{119}}{2}$$

$$=-\frac{1}{2}+\frac{\sqrt{121}}{2}=\frac{11-1}{2}=5$$

24. 다음 중 $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42}$ 을 간단히 한 것은?

- ① $\frac{2}{13}$
- ② $\frac{4}{13}$
- ③ $\frac{5}{14}$
- ④ $\frac{23}{30}$
- ⑤ $\frac{31}{42}$

해설

$$\begin{aligned} &\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} \\ &= \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} \\ &= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) \\ &= \frac{1}{2} - \frac{1}{7} = \frac{5}{14} \end{aligned}$$

25. 분수식 $2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \dots}}}$ 의 값을 구하면?

- ① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{4}{5}$

해설

(준식) = a 라 하면

$$2 - \frac{1}{a} = a \rightarrow a^2 - 2a + 1 = 0 \rightarrow (a - 1)^2 = 0$$

$$\therefore a = 1$$

26. $1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} = 5$ 을 만족하는 x 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\begin{aligned} 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} &= 1 - \frac{x-1}{x-1-x} \\ &= 1+x-1=x \\ \therefore x &= 5 \end{aligned}$$

27. 분수식 $\frac{1}{1 - \frac{1}{1 - \frac{1}{a+1}}}$ 을 간단히 하면?

① $-a$

② a

③ $a-1$

④ $1-a$

⑤ $2a-1$

해설

밑에서부터 계산해 올라간다.

$$\begin{aligned}\frac{1}{1 - \frac{1}{1 - \frac{1}{a+1}}} &= \frac{1}{1 - \frac{1}{\frac{a}{a+1}}} = \frac{1}{1 - \frac{a+1}{a}} \\ &= \frac{1}{\frac{-1}{a}} = -a\end{aligned}$$