1. $\dfrac{x-2}{2x^2-5x+3}+\dfrac{3x-1}{2x^2+x-6}+\dfrac{2x^2-5}{x^2+x-2}$ 을 계산하여라.

▶ 답 :

▷ 정답 : 2

해설

(준 식)

$$=\frac{x-2}{(2x-3)(x-1)}+\frac{3x-1}{(2x-3)(x+2)}+\frac{2x^2-5}{x^2+x-2}$$

$$=\frac{(x-2)(x+2)+(3x-1)(x-1)}{(2x-3)(x-1)(x+2)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$=\frac{4x^2-4x-3}{(2x-3)(x-1)(x+2)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$=\frac{(2x-3)(2x+1)}{(2x-3)(x+2)(x-1)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$=\frac{2x+1}{(x+2)(x-1)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$=\frac{2x^2+2x-4}{(x+2)(x-1)}=2$$

2. 분수식 $\dfrac{1}{x^2+x-2}-\dfrac{x+1}{x^2-4x+3}\div\dfrac{2x^2+5x+3}{x^2-5x+6}$ 을 간단히 하면 ?

① 1

② -2

③ $\dfrac{-x^2+2x+7}{(x-1)(x+2)(2x+3)}$

④ $\dfrac{x^2-2x+7}{(x-1)(x+2)(2x+3)}$

⑤ $\dfrac{-x^2+2x+7}{(x+1)(x-2)(2x-3)}$

해설

$$\frac{1}{x^2+x-2}-\frac{x+1}{x^2-4x+3}\div\frac{2x^2+5x+3}{x^2-5x+6}$$

$$=\frac{1}{(x+2)(x-1)}-\frac{x+1}{(x-3)(x-1)}$$

$$\times\frac{(x-2)(x-3)}{(2x+3)(x+1)}$$

$$=\frac{2x+3-(x-2)(x+2)}{(x+2)(x-1)(2x+3)}$$

$$=\frac{-x^2+2x+7}{(x-1)(x+2)(2x+3)}$$

3. $\dfrac{x+1}{x(x-1)} = \dfrac{a}{x} + \dfrac{b}{x-1}$ 가 x 에 대한 항등식일 때, 상수 a^2+b^2 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\frac{x+1}{x(x-1)} = \frac{(a+b)x-a}{x(x-1)}$$

따라서, $a+b=1$, $a=-1$

$\therefore\ a=-1,\ b=2$

$\therefore a^2+b^2=(-1)^2+2^2=5$

4. 분수식 $\dfrac{1}{x(x+1)}+\dfrac{1}{(x+1)(x+2)}$을 간단히 하면?

① $\dfrac{2}{x(x+1)}$ ② $\dfrac{1}{x(x+2)}$ ③ $\dfrac{1}{x(x+1)}$

④ $\dfrac{2}{x(x+2)}$ ⑤ $\dfrac{3}{x(x+2)}$

해설

$$\frac{1}{x(x+1)}=\frac{1}{(x+1)-x}\left(\frac{1}{x}-\frac{1}{x+1}\right)$$

$$=\frac{1}{x}-\frac{1}{x+1}$$

$$\frac{1}{(x+1)(x+2)}$$

$$=\frac{1}{(x+2)-(x+1)}\left(\frac{1}{x+1}-\frac{1}{x+2}\right)$$

$$=\frac{1}{x+1}-\frac{1}{x+2}$$

$$\therefore (\text{준식})=\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x+1}-\frac{1}{x+2}$$

$$=\frac{1}{x}-\frac{1}{x+2}=\frac{2}{x(x+2)}$$

5. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{99\cdot 100}$ 을 간단히 하면?

① $\frac{98}{99}$ ② $\frac{100}{99}$ ③ $\frac{99}{100}$ ④ $\frac{101}{100}$ ⑤ $\frac{100}{101}$

해설

이항분리 이용

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{99\cdot 100}$$
$$=\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{99}-\frac{1}{100}$$
$$=1-\frac{1}{100}=\frac{99}{100}$$

6. 다음 식을 간단히 한 식은?

$$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{a}}}$$

① $a+1$ ② $a+2$ ③ $-a+1$

④ $-a+2$ ⑤ $a-1$

해설

아래에서부터 계산해 올라가자.

$$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{a}}} = \cfrac{1}{1-\cfrac{a}{a-1}} = \cfrac{a-1}{a-1-a} = -a+1$$

7. $\dfrac{2^1+2^0+2^{-1}}{2^{-2}+2^{-3}+2^{-4}}$ 를 풀면?

① 6 ② 8 ③ $\dfrac{31}{2}$ ④ 24 ⑤ 512

해설

분자, 분모에 2^3 을 곱하면

$$\frac{2^3(2^1+2^0+2^{-1})}{2^3(2^{-2}+2^{-3}+2^{-4})} = \frac{2^3(2^1+2^0+2^{-1})}{2^1+2^0+2^{-1}}$$

$$= 2^3 = 8$$

해설

$$\frac{2+1+\dfrac{1}{2}}{\dfrac{1}{2^2}+\dfrac{1}{2^3}+\dfrac{1}{2^4}} = \frac{\dfrac{7}{2}}{\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}} = \frac{\dfrac{7}{2}}{\dfrac{7}{16}} = 8$$

8. $\frac{x^3-x}{x^2-x}+\frac{x^4-1}{x^2-1}-\frac{x^2-2x-3}{x+1}\times\frac{x+2}{x^2-x-6}$ 을 계산하면?

① x^2+x+1 ② $\frac{x^2+1}{x-1}$ ③ $\frac{2x}{x^2-1}$

④ x^2-1 ⑤ $\frac{2x-1}{x^2-x}$

해설

$$\frac{x(x+1)(x-1)}{x(x-1)}+\frac{(x^2+1)(x^2-1)}{x^2-1}$$

$$-\frac{(x+1)(x-3)}{x+1}\times\frac{x+2}{(x-3)(x+2)}$$

$$=x+1+x^2+1-1=x^2+x+1$$

9. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$을 간단히 하면?

① $\frac{5}{12}$ ② $\frac{1}{2}$ ③ $\frac{11}{20}$ ④ $\frac{2}{3}$ ⑤ $\frac{7}{10}$

해설

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$$

$$=\frac{1\cdot 3}{2^2}\cdot\frac{2\cdot 4}{3^2}\cdots\frac{8\cdot 10}{9^2}\cdot\frac{9\cdot 11}{10^2}$$

$$=\frac{1\cdot 11}{2\cdot 10}=\frac{11}{20}$$

일반적으로

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

10. $\dfrac{1}{a(a+1)}+\dfrac{2}{(a+1)(a+3)}+\dfrac{3}{(a+3)(a+6)}$ 을 간단히 한 것은 ?

① $\dfrac{1}{a}+\dfrac{6}{a+6}$ ② $\dfrac{1}{a}+\dfrac{1}{a+6}$ ③ $\dfrac{1}{a}-\dfrac{1}{a+6}$

④ $\dfrac{1}{a}-\dfrac{6}{a+6}$ ⑤ $\dfrac{2}{a}-\dfrac{1}{a+6}$

해설

(준식)

$=\dfrac{1}{a}-\dfrac{1}{a+1}+\dfrac{1}{a+1}-\dfrac{1}{a+3}+\dfrac{1}{a+3}-\dfrac{1}{a+6}$

$=\dfrac{1}{a}-\dfrac{1}{a+6}$

11. 분수식 $\dfrac{x^2}{(x-y)(x-z)}+\dfrac{y^2}{(y-x)(y-z)}+\dfrac{z^2}{(z-x)(z-y)}$ 를 간단히 하여라.

▶ 답 :

▷ 정답 : 1

해설

$$\frac{x^2(z-y)+y^2(z-x)+z^2(y-x)}{(x-y)(y-z)(z-x)} \cdots ①$$

①에서 분자를 x에 관하여 정리하면

$$\begin{aligned}
&x^2(z-y)+y^2(z-x)+z^2(y-x)\\
&=(z-y)x^2-(z^2-y^2)x+yz^2-y^2z\\
&=(z-y)x^2-(z+y)(z-y)x+zy(z-y)\\
&=(z-y)\{x^2-(z+y)x+zy\}\\
&=(z-y)(x-z)(x-y)=(x-y)(y-z)(z-x)
\end{aligned}$$

$$\therefore \text{(준식)}=\frac{(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}=1$$

12. 다음 식을 간단히 하면?

$$\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}-\frac{4}{1+x^4}$$

① $\frac{8x^4}{1-x^4}$ ② $\frac{8}{1-x^4}$ ③ $\frac{8x^4}{1-x^8}$

④ $\frac{8}{1-x^8}$ ⑤ $\frac{8x^4}{1+x^8}$

해설

$$\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}-\frac{4}{1+x^4}$$

$$=\frac{2}{1-x^2}+\frac{2}{1+x^2}-\frac{4}{1+x^4}$$

$$=\frac{4}{1-x^4}-\frac{4}{1+x^4}=\frac{8x^4}{1-x^8}$$

13. $x = \frac{a}{b}$, $a \neq b$, $b \neq 0$ 일 때, $\frac{a+b}{a-b}$ 는?

① $\frac{x}{x+1}$ ② $\frac{x+1}{x-1}$ ③ 1 ④ $x - \frac{1}{x}$ ⑤ $x + \frac{1}{x}$

해설

$a = bx$ 이므로 $\frac{a+b}{a-b} = \frac{bx+b}{bx-b} = \frac{x+1}{x-1}$

또는 $\frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{x+1}{x-1}$

14. $\dfrac{x+3}{(x+1)(x+2)} = \dfrac{a}{x+1} + \dfrac{b}{x+2}$ 을 만족할 때, $a^2 + b^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\frac{x+3}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}$$

$$= \frac{(a+b)\,x + 2a + b}{(x+1)\,(x+2)}$$

$a + b = 1, 2a + b = 3$

$\therefore\ a = 2,\ b = -1$

$\therefore\ a^2 + b^2 = 2^2 + (-1)^2 = 5$

15. 등식 $\dfrac{x^2+1}{x^3-6x^2+11x-6}=\dfrac{a}{x-1}+\dfrac{b}{x-2}+\dfrac{c}{x-3}$ 이 x에 대한 항등식이 되도록 상수 $a,\ b,\ c$에 대하여 abc의 값을 구하여라.

▶ 답:

▷ 정답: -25

해설

$x^3-6x^2+11x-6=(x-1)(x-2)(x-3)$ 이므로
양변에 $(x-1)(x-2)(x-3)$을 곱하면
$x^2+1=a(x-2)(x-3)+b(x-1)(x-3)+c(x-1)(x-2)$
양변에 $x=1$을 대입하면 $2=2a$
$\therefore\ a=1$
양변에 $x=2$를 대입하면 $5=-b$
$\therefore\ b=-5$
양변에 $x=3$을 대입하면 $10=2c$
$\therefore\ c=5$
$\therefore\ abc=-25$

16. 등식 $\frac{a}{x}-\frac{b}{x+1}=\frac{1}{x(x+1)}$ 이 x에 대한 항등식일 때, 상수 a, b의 합을 구하면?

① 0 ② 1 ③ 2 ④ 3 ⑤ 4

해설

양변에 $x(x+1)$을 곱하면

$a(x+1)-bx=1,\ ax+a-bx-1=0$

$x(a-b)+(a-1)=0$

이 등식이 x에 대한 항등식이므로

$a=b,\ a=1,\ b=1$

$\therefore\ a+b=2$

17. 분수식 $\frac{x}{x+1}+\frac{2x-1}{x-1}-\frac{3x^2+4x+2}{x^2+x}$를 간단히 하면?

① $-\frac{x-2}{x(x-1)}$ ② $\frac{x+2}{x(x+1)}$ ③ $\frac{x-2}{x(x+1)}$
④ $\frac{x+2}{x(x-1)}$ ⑤ $\frac{x-2}{x(x-1)}$

해설

$\frac{x}{x+1}=1-\frac{1}{x+1}$,
$\frac{2x-1}{x-1}=2+\frac{1}{x-1}$,
$\frac{3x^2+4x+2}{x^2+x}=3+\frac{x+2}{x^2+x}$ 이므로

$$(\text{준식})=\left(1-\frac{1}{x+1}\right)+\left(2+\frac{1}{x-1}\right)-\left(3+\frac{x+2}{x^2+x}\right)$$

$$=-\frac{1}{x+1}+\frac{1}{x-1}-\frac{x+2}{x(x+1)}$$

$$=\frac{-x(x-1)+x(x+1)-(x-1)(x+2)}{x(x+1)(x-1)}$$

$$=\frac{-x^2+x+x^2+x-x^2-x+2}{x(x+1)(x-1)}$$

$$=\frac{-x^2+x+2}{x(x-1)(x+1)}$$

$$=\frac{-(x^2-x-2)}{x(x-1)(x+1)}$$

$$=\frac{-(x-2)(x+1)}{x(x-1)(x+1)}$$

$$=-\frac{x-2}{x(x-1)}$$

18. $\dfrac{x+2}{x+1}-\dfrac{x+3}{x+2}-\dfrac{x+4}{x+3}+\dfrac{x+5}{x+4}$를 간단히 하면?

① $\dfrac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}$

② $\dfrac{2}{(x+1)(x+2)(x+3)(x+4)}$

③ $\dfrac{2x}{(x+1)(x+2)(x+3)(x+4)}$

④ $\dfrac{2(x-1)}{(x+1)(x+2)(x+3)(x+4)}$

⑤ $\dfrac{2(x-2)}{(x+1)(x+2)(x+3)(x+4)}$

해설

$$
\begin{aligned}
(\text{준 식}) &= \left(1+\frac{1}{x+1}\right)-\left(1+\frac{1}{x+2}\right) \\
&\quad -\left(1+\frac{1}{x+3}\right)+\left(1+\frac{1}{x+4}\right) \\
&= \frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{x+3}+\frac{1}{x+4} \\
&= \frac{2x+5}{(x+1)(x+4)}-\frac{2x+5}{(x+2)(x+3)} \\
&= \frac{(2x+5)(x^2+5x+6-x^2-5x-4)}{(x+1)(x+2)(x+3)(x+4)} \\
&= \frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}
\end{aligned}
$$

19. 부분분수를 이용하여 다음을 만족시키는 양수 x 를 구하여라.

$$\frac{1}{x(x+2)}+\frac{1}{(x+2)(x+4)}$$
$$+\frac{1}{(x+4)(x+6)}+\frac{1}{(x+6)(x+8)}=\frac{4}{9}$$

▶ 답:

▷ 정답: 1

해설

주어진 식을 부분분수로 나타내면

$$\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+2}\right)+\frac{1}{2}\left(\frac{1}{x+2}-\frac{1}{x+4}\right)$$
$$+\frac{1}{2}\left(\frac{1}{x+4}-\frac{1}{x+6}\right)+\frac{1}{2}\left(\frac{1}{x+6}-\frac{1}{x+8}\right)$$
$$=\frac{1}{2}\left\{\left(\frac{1}{x}-\frac{1}{x+2}\right)+\left(\frac{1}{x+2}-\frac{1}{x+4}\right)\right.$$
$$\left.+\left(\frac{1}{x+4}-\frac{1}{x+6}\right)+\left(\frac{1}{x+6}-\frac{1}{x+8}\right)\right\}$$
$$=\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+8}\right)=\frac{1}{2}\cdot\frac{8}{x(x+8)}=\frac{4}{x(x+8)}$$
$$=\frac{4}{9}$$
$\therefore x(x+8)=9$
$x^2+8x-9=(x-1)(x+9)=0$
$x>0$ 이므로 $x=1$

20. $\dfrac{1}{x(x+1)}+\dfrac{x}{(x+1)(2x+1)}+\dfrac{x}{(2x+1)(3x+1)}-\dfrac{2x+1}{x(3x+1)}$ 을 간단히 하면 ?

① -2 ② -1 ③ 0 ④ 1 ⑤ $\dfrac{2}{3}$

해설

주어진 식을 이항분리시키면,

$$\begin{aligned}(\text{준식}) &= \left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{2x+1}\right)\\ &\quad+\left(\frac{1}{2x+1}-\frac{1}{3x+1}\right)-\left(\frac{1}{x}-\frac{1}{3x+1}\right)\\ &= 0\end{aligned}$$

21. 분수식

$$\frac{1}{x(x+1)}+\frac{2}{(x+1)(x+3)}+\frac{3}{(x+3)(x+6)}$$
$$+\frac{4}{(x+6)(x+10)}+\frac{5}{(x+10)(x+15)}$$
$$+\frac{6}{(x+15)(x+21)}+\frac{7}{(x+21)(x+28)}$$ 을 간단히 하면?

① $\dfrac{28}{x^2+28x}$ ② $\dfrac{7}{x^2+28x}$ ③ $\dfrac{7}{x^2+21x}$

④ $\dfrac{6}{x^2+21x}$ ⑤ $\dfrac{1}{x^2+21x}$

해설

$$(\text{준식}) = \left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{x+3}\right)+\cdots$$
$$+\left(\frac{1}{x+21}-\frac{1}{x+28}\right)$$
$$=\frac{1}{x}-\frac{1}{x+28}=\frac{28}{x^2+28x}$$

22. $\frac{1}{2\cdot 4}+\frac{1}{4\cdot 6}+\frac{1}{6\cdot 8}+\cdots+\frac{1}{18\cdot 20}$ 을 계산한 값은?

① 0 ② $\frac{9}{20}$ ③ 40 ④ $\frac{40}{9}$ ⑤ $\frac{9}{40}$

해설

$$\frac{1}{2}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{18}-\frac{1}{20}\right)\right\}$$

$$=\frac{1}{2}\left\{\frac{1}{2}+\left(-\frac{1}{4}+\frac{1}{4}\right)+\left(-\frac{1}{6}+\frac{1}{6}\right)+\cdots-\frac{1}{20}\right\}$$

$$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{20}\right)=\frac{1}{2}\cdot\frac{9}{20}=\frac{9}{40}$$

23. 다음의 식을 간단히 하면?

$$\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{119}+\sqrt{121}}$$

① 5 ② 10 ③ 0 ④ −10 ⑤ −5

해설

준식을 유리화하면

$$\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\cdots+\frac{\sqrt{121}-\sqrt{119}}{2}$$

$$=-\frac{1}{2}+\frac{\sqrt{121}}{2}=\frac{11-1}{2}=5$$

24. 다음 중 $\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$을 간단히 한 것은?

① $\frac{2}{13}$ ② $\frac{4}{13}$ ③ $\frac{5}{14}$ ④ $\frac{23}{30}$ ⑤ $\frac{31}{42}$

해설

$$\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$$
$$=\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+\frac{1}{6\times7}$$
$$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)$$
$$=\frac{1}{2}-\frac{1}{7}=\frac{5}{14}$$

25. 분수식 $2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-\cdots}}}$ 의 값을 구하면?

① $\frac{1}{2}$ ② 1 ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{4}{5}$

해설

(준식)$=a$라 하면

$$2-\frac{1}{a}=a \;\rightarrow\; a^2-2a+1=0 \;\rightarrow\; (a-1)^2=0$$

$\therefore\ a=1$

26. $1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{x}}}=5$을 만족하는 x의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{x}}}=1-\frac{x-1}{x-1-x}$$

$$=1+x-1=x$$

$\therefore x=5$

27. 분수식 $\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{a+1}}}$ 을 간단히 하면?

① $-a$ ② a ③ $a-1$

④ $1-a$ ⑤ $2a-1$

해설

밑에서부터 계산해 올라간다.

$$\frac{1}{1-\dfrac{1}{1-\dfrac{1}{a+1}}} = \frac{1}{1-\dfrac{1}{\dfrac{a}{a+1}}} = \frac{1}{1-\dfrac{a+1}{a}}$$

$$= \frac{1}{\dfrac{-1}{a}} = -a$$