1. $\dfrac{x}{x^2 - xy} + \dfrac{y}{y^2 - xy}$ 을 계산하여라.

▶ **답 :**

▷ **정답 :** 0

해설

(준식) $= \dfrac{x}{x(x-y)} + \dfrac{y}{y(y-x)} = \dfrac{1}{x-y} + \dfrac{-1}{x-y}$

$= 0$

2. $\dfrac{x^2 - 2x}{x^2 - 4x + 4} \times \dfrac{x^2 - 7x + 10}{x^2 - 5x}$ 을 계산하여라.

▶ **답 :**

▷ **정답 :** 1

해설

(준식)$= \dfrac{x(x-2)}{(x-2)^2} \times \dfrac{(x-2)(x-5)}{x(x-5)} = 1$

3. $\dfrac{x-2}{2x^2-5x+3}+\dfrac{3x-1}{2x^2+x-6}+\dfrac{2x^2-5}{x^2+x-2}$ 을 계산하여라.

▶ 답 :

▷ 정답 : 2

해설

(준 식)

$$= \frac{x-2}{(2x-3)(x-1)}+\frac{3x-1}{(2x-3)(x+2)}+\frac{2x^2-5}{x^2+x-2}$$

$$= \frac{(x-2)(x+2)+(3x-1)(x-1)}{(2x-3)(x-1)(x+2)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$= \frac{4x^2-4x-3}{(2x-3)(x-1)(x+2)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$= \frac{(2x-3)(2x+1)}{(2x-3)(x+2)(x-1)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$= \frac{2x+1}{(x+2)(x-1)}+\frac{2x^2-5}{(x+2)(x-1)}$$

$$= \frac{2x^2+2x-4}{(x+2)(x-1)} = 2$$

4. $\frac{x-1}{3x-6} \times \frac{2x-4}{x^2-x}$ 를 계산하시오.

▶ 답 :

▷ 정답 : $\frac{2}{3x}$

해설

$$\frac{x-1}{3x-6} \times \frac{2x-4}{x^2-x} = \frac{2(x-1)(x-2)}{3x(x-2)(x-1)} = \frac{2}{3x}$$

5. $x^2 \neq 4$인 모든 실수 x에 대하여 $\dfrac{x+6}{x^2-4} = \dfrac{a}{x+2} - \dfrac{b}{x-2}$을 만족시키는 상수 a와 b가 있다. 이때, $a+b$의 값은?

① -6　　[✓ ②] -3　　③ -1　　④ 2　　⑤ 4

해설

$\dfrac{x+6}{x^2-4} = \dfrac{a}{x+2} - \dfrac{b}{x-2}$의 우변을 통분하여 계산하면

$$\begin{aligned}\frac{a}{x+2} - \frac{b}{x-2} &= \frac{a(x-2)}{x^2-4} - \frac{b(x+2)}{x^2-4} \\ &= \frac{(a-b)x-2(a+b)}{x^2-4}\end{aligned}$$

따라서 $a-b=1,\ -2(a+b)=6$

$\therefore\ a=-1,\ b=-2$

$\therefore\ a+b=-1-2=-3$

6. $\dfrac{1}{x(x+1)}+\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}$ 을 간단히 하면?

① $\dfrac{2}{x(x+2)}$ ② $\dfrac{3}{x(x+2)}$

③ $\dfrac{2}{(x+2)(x+3)}$ ④ $\dfrac{3}{(x+2)(x+3)}$

[✓ ⑤ $\dfrac{3}{x(x+3)}$]

해설

$$\begin{aligned}(\text{준식}) &= \left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{x+2}\right)\\ &\quad+\left(\frac{1}{x+2}-\frac{1}{x+3}\right)\\ &= \frac{1}{x}-\frac{1}{x+3}=\frac{3}{x(x+3)}\end{aligned}$$

7. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{99\cdot 100}$ 을 간단히 하면?

① $\frac{98}{99}$ ② $\frac{100}{99}$ [✓ ③ $\frac{99}{100}$] ④ $\frac{101}{100}$ ⑤ $\frac{100}{101}$

해설

이항분리 이용

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{99\cdot 100}$$

$$=\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{99}-\frac{1}{100}$$

$$=1-\frac{1}{100}=\frac{99}{100}$$

8. 다음 식을 간단히 한 식은?

$$\frac{1}{1-\frac{1}{1-\frac{1}{a}}}$$

① $a+1$
② $a+2$
[✓ ③ $-a+1$]
④ $-a+2$
⑤ $a-1$

해설

아래에서부터 계산해 올라가자.

$$\frac{1}{1-\frac{1}{1-\frac{1}{a}}}=\frac{1}{1-\frac{a}{a-1}}=\frac{a-1}{a-1-a}=-a+1$$

9. $x^2-5x+1=0$일 때, $x^2+\dfrac{1}{x^2}$의 값을 구하시오.

▶ 답 :

▷ 정답 : 23

해설

$x^2-5x+1=0$에서 양변을 x로 나눈다.

$x+\dfrac{1}{x}-5=0$

$x+\dfrac{1}{x}=5$

$x^2+\dfrac{1}{x^2}=\left(x+\dfrac{1}{x}\right)^2-2=25-2=23$

10. $\dfrac{2^1+2^0+2^{-1}}{2^{-2}+2^{-3}+2^{-4}}$ 를 풀면?

① 6 [✓ ② 8] ③ $\dfrac{31}{2}$ ④ 24 ⑤ 512

해설

분자, 분모에 2^3 을 곱하면

$$\frac{2^3(2^1+2^0+2^{-1})}{2^3(2^{-2}+2^{-3}+2^{-4})} = \frac{2^3(2^1+2^0+2^{-1})}{2^1+2^0+2^{-1}}$$
$$= 2^3 = 8$$

해설

$$\frac{2+1+\frac{1}{2}}{\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}} = \frac{\frac{7}{2}}{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} = \frac{\frac{7}{2}}{\frac{7}{16}} = 8$$

11. $\dfrac{x^3-x}{x^2-x}+\dfrac{x^4-1}{x^2-1}-\dfrac{x^2-2x-3}{x+1}\times\dfrac{x+2}{x^2-x-6}$ 을 계산하면?

[✓ ①] x^2+x+1　② $\dfrac{x^2+1}{x-1}$　③ $\dfrac{2x}{x^2-1}$

④ x^2-1　⑤ $\dfrac{2x-1}{x^2-x}$

해설

$$\frac{x(x+1)(x-1)}{x(x-1)}+\frac{(x^2+1)(x^2-1)}{x^2-1}$$

$$-\frac{(x+1)(x-3)}{x+1}\times\frac{x+2}{(x-3)(x+2)}$$

$$=x+1+x^2+1-1=x^2+x+1$$

12. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$을 간단히 하면?

① $\frac{5}{12}$ ② $\frac{1}{2}$ [✓ ③ $\frac{11}{20}$] ④ $\frac{2}{3}$ ⑤ $\frac{7}{10}$

해설

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$$

$$=\frac{1\cdot 3}{2^2}\cdot\frac{2\cdot 4}{3^2}\cdots\frac{8\cdot 10}{9^2}\cdot\frac{9\cdot 11}{10^2}$$

$$=\frac{1\cdot 11}{2\cdot 10}=\frac{11}{20}$$

일반적으로

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

13. $\frac{1}{a(a+1)}+\frac{2}{(a+1)(a+3)}+\frac{3}{(a+3)(a+6)}$ 을 간단히 한 것은 ?

① $\frac{1}{a}+\frac{6}{a+6}$
② $\frac{1}{a}+\frac{1}{a+6}$
[✓ ③ $\frac{1}{a}-\frac{1}{a+6}$]
④ $\frac{1}{a}-\frac{6}{a+6}$
⑤ $\frac{2}{a}-\frac{1}{a+6}$

해설

(준식)

$=\frac{1}{a}-\frac{1}{a+1}+\frac{1}{a+1}-\frac{1}{a+3}+\frac{1}{a+3}-\frac{1}{a+6}$

$=\frac{1}{a}-\frac{1}{a+6}$

14. 분수식 $\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-x)(y-z)}+\frac{z^2}{(z-x)(z-y)}$를 간단히 하여라.

▶ 답 :

▷ 정답 : 1

해설

$\frac{x^2(z-y)+y^2(z-x)+z^2(y-x)}{(x-y)(y-z)(z-x)}\cdots$①

①에서 분자를 x에 관하여 정리하면

$x^2(z-y)+y^2(z-x)+z^2(y-x)$

$=(z-y)x^2-(z^2-y^2)x+yz^2-y^2z$

$=(z-y)x^2-(z+y)(z-y)x+zy(z-y)$

$=(z-y)\left\{x^2-(z+y)x+zy\right\}$

$=(z-y)(x-z)(x-y)=(x-y)(y-z)(z-x)$

$\therefore$ (준식) $=\frac{(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}=1$

15. 분수식 $\frac{2x}{x+1}+\frac{3x-2}{x-1}-\frac{5x^2-7x+3}{x^2-x}$ 을 간단히 하면?

① $-\frac{x-3}{x(x+1)}$ ② $\frac{x-3}{x(x+1)}$ [✓ ③ $\frac{x+3}{x(x+1)}$]
④ $\frac{x+3}{x(x-1)}$ ⑤ $\frac{x-3}{x(x-1)}$

해설

(준식)

$$=\frac{2x(x-1)+(3x-2)(x+1)}{(x-1)(x+1)}-\frac{5x^2-7x+3}{x(x-1)}$$

$$=\frac{5x^2-x-2}{(x-1)(x+1)}-\frac{5x^2-7x+3}{x(x-1)}$$

$$=\frac{(5x^3-x^2-2x)-(5x^2-7x+3)(x+1)}{x(x-1)(x+1)}$$

$$=\frac{x^2+2x-3}{x(x-1)(x+1)}$$

$$=\frac{(x+3)(x-1)}{x(x-1)(x+1)}=\frac{x+3}{x(x+1)}$$

해설

분자의 차수를 줄여서 계산할 수 있다.

$$\frac{2x}{x+1}+\frac{3x-2}{x-1}-\frac{5x^2-7x+3}{x^2-x}$$

$$=\frac{2(x+1)-2}{x+1}+\frac{3(x-1)+1}{x-1}-\frac{5(x^2-x)-2x+3}{x^2-x}$$

$$=(2+3-5)+\frac{-2}{x+1}+\frac{1}{x-1}+\frac{2x-3}{x(x-1)}$$

$$=\frac{-2(x-1)+x+1}{(x+1)(x-1)}+\frac{2x-3}{x(x-1)}$$

$$=\frac{x(-x+3)+(2x-3)(x+1)}{x(x+1)(x-1)}$$

$$=\frac{(x-1)(x+3)}{x(x+1)(x-1)}=\frac{x+3}{x(x+1)}$$

16. 다음 식을 간단히 하면?

$$\boxed{\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}-\frac{4}{1+x^4}}$$

① $\frac{8x^4}{1-x^4}$ ② $\frac{8}{1-x^4}$ [✓ ③ $\frac{8x^4}{1-x^8}$]

④ $\frac{8}{1-x^8}$ ⑤ $\frac{8x^4}{1+x^8}$

해설

$$\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}-\frac{4}{1+x^4}$$

$$=\frac{2}{1-x^2}+\frac{2}{1+x^2}-\frac{4}{1+x^4}$$

$$=\frac{4}{1-x^4}-\frac{4}{1+x^4}=\frac{8x^4}{1-x^8}$$

17. $x = \frac{a}{b}$, $a \neq b$, $b \neq 0$ 일 때, $\frac{a+b}{a-b}$ 는?

① $\frac{x}{x+1}$ [✓ ② $\frac{x+1}{x-1}$] ③ 1 ④ $x - \frac{1}{x}$ ⑤ $x + \frac{1}{x}$

해설

$a = bx$이므로 $\frac{a+b}{a-b} = \frac{bx+b}{bx-b} = \frac{x+1}{x-1}$

또는 $\frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{x+1}{x-1}$

18. $\frac{x+3}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}$ 을 만족할 때, $a^2 + b^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$$\frac{x+3}{(x+1)(x+2)} = \frac{a}{x+1} + \frac{b}{x+2}$$
$$= \frac{(a+b)\,x + 2a + b}{(x+1)\,(x+2)}$$

$a + b = 1, 2a + b = 3$

$\therefore\ a = 2,\ b = -1$

$\therefore\ a^2 + b^2 = 2^2 + (-1)^2 = 5$

19. 등식 $\frac{3x}{x^3+1} = \frac{a}{x+1} + \frac{bx+c}{x^2-x+1}$ 가 x에 대한 항등식이 되도록 상수 $a,\ b,\ c$의 값을 정할 때, $a+b+c$의 값은?

① -3　② -1　③ 0　[✓ ④ 1]　⑤ 3

해설

$$(\text{우변}) = \frac{a(x^2-x+1)+(bx+c)(x+1)}{(x+1)(x^2-x+1)}$$

$$= \frac{(a+b)x^2+(-a+b+c)x+a+c}{x^3+1}$$

주어진 등식이 x에 대한 항등식이 되려면

$a+b=0,\ -a+b+c=3,\ a+c=0$

이것을 풀면

$a=-1,\ b=1,\ c=1$

$\therefore\ a+b+c=1$

20. 등식 $\frac{3x}{x^3+1}=\frac{a}{x+1}+\frac{bx+c}{x^2-x+1}$ 가 x에 관한 항등식일 때, $a+b+c$ 의 값은?

① -2 ② -6 [✓ ③ 1] ④ 2 ⑤ $\frac{7}{4}$

해설

$\frac{3x}{x^3+1}=\frac{a}{x+1}+\frac{bx+c}{x^2-x+1}$

$=\frac{a(x^2-x+1)+(x+1)(bx+c)}{x^3+1}$

$=\frac{ax^2-ax+a+bx^2+bx+cx+c}{x^3+1}$

$=\frac{(a+b)x^2+(b-a+c)x+a+c}{x^3+1}$

$a+b=0,\ b-a+c=3,\ a+c=0$을 연립하여 풀면

$a=-1,\ b=1,\ c=1$

$\therefore\ a+b+c=1$

21. $\dfrac{4x^2}{(x-1)^2(x+1)} = \dfrac{a}{x-1} + \dfrac{b}{(x-1)^2} + \dfrac{c}{x+1}$ 가 x 에 관한 항등식이 되도록 실수 $a,\ b,\ c$ 의 값을 정하였을 때, abc 의 값은?

① 2　② 3　[✓ ③ 6]　④ 12　⑤ 24

해설

$\dfrac{4x^2}{(x-1)^2(x+1)} = \dfrac{a}{x-1} + \dfrac{b}{(x-1)^2} + \dfrac{c}{x+1} \cdots$①
①의 양변에 $(x-1)^2(x+1)$을 곱하면
$4x^2 = a(x-1)(x+1) + b(x+1) + c(x-1)^2 \cdots$②
②가 x에 관한 항등식이므로
$x = 1, -1, 0$을 각각 대입하면
$4 = 2b,\ 4 = 4c,\ 0 = -a + b + c$
$\therefore\ a = 3,\ b = 2,\ c = 1$
$\therefore\ abc = 6$

22. 분수식 $\frac{x}{x+1}+\frac{2x-1}{x-1}-\frac{3x^2+4x+2}{x^2+x}$를 간단히 하면?

[✓ ① $-\frac{x-2}{x(x-1)}$]　② $\frac{x+2}{x(x+1)}$　③ $\frac{x-2}{x(x+1)}$

④ $\frac{x+2}{x(x-1)}$　⑤ $\frac{x-2}{x(x-1)}$

해설

$\frac{x}{x+1}=1-\frac{1}{x+1}$,

$\frac{2x-1}{x-1}=2+\frac{1}{x-1}$,

$\frac{3x^2+4x+2}{x^2+x}=3+\frac{x+2}{x^2+x}$ 이므로

$$(\text{준식})=\left(1-\frac{1}{x+1}\right)+\left(2+\frac{1}{x-1}\right)$$

$$-\left(3+\frac{x+2}{x^2+x}\right)$$

$$=-\frac{1}{x+1}+\frac{1}{x-1}-\frac{x+2}{x(x+1)}$$

$$=\frac{-x(x-1)+x(x+1)-(x-1)(x+2)}{x(x+1)(x-1)}$$

$$=\frac{-x^2+x+x^2+x-x^2-x+2}{x(x+1)(x-1)}$$

$$=\frac{-x^2+x+2}{x(x-1)(x+1)}$$

$$=\frac{-(x^2-x-2)}{x(x-1)(x+1)}$$

$$=\frac{-(x-2)(x+1)}{x(x-1)(x+1)}$$

$$=-\frac{x-2}{x(x-1)}$$

23. $\frac{x+2}{x+1} - \frac{x+3}{x+2} - \frac{x+4}{x+3} + \frac{x+5}{x+4}$ 를 간단히 하면?

[✓ ①] $\frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}$

② $\frac{2}{(x+1)(x+2)(x+3)(x+4)}$

③ $\frac{2x}{(x+1)(x+2)(x+3)(x+4)}$

④ $\frac{2(x-1)}{(x+1)(x+2)(x+3)(x+4)}$

⑤ $\frac{2(x-2)}{(x+1)(x+2)(x+3)(x+4)}$

해설

$$
\begin{aligned}
(\text{준 식}) &= \left(1+\frac{1}{x+1}\right) - \left(1+\frac{1}{x+2}\right) \\
&\quad - \left(1+\frac{1}{x+3}\right) + \left(1+\frac{1}{x+4}\right) \\
&= \frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{x+3} + \frac{1}{x+4} \\
&= \frac{2x+5}{(x+1)(x+4)} - \frac{2x+5}{(x+2)(x+3)} \\
&= \frac{(2x+5)(x^2+5x+6-x^2-5x-4)}{(x+1)(x+2)(x+3)(x+4)} \\
&= \frac{2(2x+5)}{(x+1)(x+2)(x+3)(x+4)}
\end{aligned}
$$

24. 부분분수를 이용하여 다음을 만족시키는 양수 x 를 구하여라.

$$\frac{1}{x(x+2)}+\frac{1}{(x+2)(x+4)}$$
$$+\frac{1}{(x+4)(x+6)}+\frac{1}{(x+6)(x+8)}=\frac{4}{9}$$

▶ **답 :**

▷ **정답 :** 1

해설

주어진 식을 부분분수로 나타내면

$$\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+2}\right)+\frac{1}{2}\left(\frac{1}{x+2}-\frac{1}{x+4}\right)$$
$$+\frac{1}{2}\left(\frac{1}{x+4}-\frac{1}{x+6}\right)+\frac{1}{2}\left(\frac{1}{x+6}-\frac{1}{x+8}\right)$$
$$=\frac{1}{2}\left\{\left(\frac{1}{x}-\frac{1}{x+2}\right)+\left(\frac{1}{x+2}-\frac{1}{x+4}\right)\right.$$
$$\left.+\left(\frac{1}{x+4}-\frac{1}{x+6}\right)+\left(\frac{1}{x+6}-\frac{1}{x+8}\right)\right\}$$
$$=\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+8}\right)=\frac{1}{2}\cdot\frac{8}{x(x+8)}=\frac{4}{x(x+8)}$$
$$=\frac{4}{9}$$

$\therefore x(x+8)=9$

$x^2+8x-9=(x-1)(x+9)=0$

$x>0$ 이므로 $x=1$

25. $x=1$일 때,

$\frac{1}{x(x+1)}+\frac{2}{(x+1)(x+3)}+\frac{3}{(x+3)(x+6)}+\frac{4}{(x+6)(x+10)}$ 의 값을 구하면?

① $\frac{8}{11}$ [✓ ② $\frac{10}{11}$] ③ $\frac{12}{11}$ ④ $\frac{8}{9}$ ⑤ $\frac{10}{9}$

해설

이항분리 이용

$$\frac{1}{x(x+1)}+\frac{2}{(x+1)(x+3)}+\frac{3}{(x+3)(x+6)}+\frac{4}{(x+6)(x+10)}$$

$$=\frac{1}{x}-\frac{1}{x+1}+\frac{2}{2}\left(\frac{1}{x+1}-\frac{1}{x+3}\right)$$

$$+\frac{3}{3}\left(\frac{1}{x+3}-\frac{1}{x+6}\right)+\frac{4}{4}\left(\frac{1}{x+6}-\frac{1}{x+10}\right)$$

$$=\frac{1}{x}-\frac{1}{x+10}$$

$x=1$ 대입하면 $\frac{1}{1}-\frac{1}{1+10}=1-\frac{1}{11}=\frac{10}{11}$

26. $\dfrac{1}{x(x+1)}+\dfrac{x}{(x+1)(2x+1)}+\dfrac{x}{(2x+1)(3x+1)}-\dfrac{2x+1}{x(3x+1)}$ 을 간단히 하면 ?

① -2　② -1　[✓ ③ 0]　④ 1　⑤ $\dfrac{2}{3}$

해설

주어진 식을 이항분리시키면,

$$\begin{aligned}(\text{준식}) &= \left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{2x+1}\right)\\ &\quad+\left(\frac{1}{2x+1}-\frac{1}{3x+1}\right)-\left(\frac{1}{x}-\frac{1}{3x+1}\right)\\ &= 0\end{aligned}$$

27. $x=4$일 때,

$\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}+\frac{1}{(x+3)(x+4)}$의 값을 구하면?

① $\frac{1}{16}$ [✓ ② $\frac{1}{8}$] ③ $\frac{1}{4}$ ④ $\frac{1}{2}$ ⑤ 1

해설

$\frac{1}{(x+a)(x+b)}=\frac{1}{b-a}\left(\frac{1}{x+a}-\frac{1}{x+b}\right)$ 이므로

$$(\text{준식})=\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x+1}-\frac{1}{x+2}+\frac{1}{x+2}$$
$$-\frac{1}{x+3}+\frac{1}{x+3}-\frac{1}{x+4}$$
$$=\frac{1}{x}-\frac{1}{x+4}$$

$\therefore\ x=4$ 대입하면 $\frac{1}{8}$

28. $\frac{1}{2\cdot 4}+\frac{1}{4\cdot 6}+\frac{1}{6\cdot 8}+\cdots+\frac{1}{18\cdot 20}$ 을 계산한 값은?

① 0　② $\frac{9}{20}$　③ 40　④ $\frac{40}{9}$　[✓ ⑤ $\frac{9}{40}$]

해설

$$\frac{1}{2}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{18}-\frac{1}{20}\right)\right\}$$

$$=\frac{1}{2}\left\{\frac{1}{2}+\left(-\frac{1}{4}+\frac{1}{4}\right)+\left(-\frac{1}{6}+\frac{1}{6}\right)+\cdots-\frac{1}{20}\right\}$$

$$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{20}\right)=\frac{1}{2}\cdot\frac{9}{20}=\frac{9}{40}$$

29. 다음의 식을 간단히 하면?

$$\boxed{\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\cdots+\frac{1}{\sqrt{119}+\sqrt{121}}}$$

[✓ ① 5]　② 10　③ 0　④ -10　⑤ -5

해설

준식을 유리화하면

$$\frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\cdots+\frac{\sqrt{121}-\sqrt{119}}{2}$$

$$=-\frac{1}{2}+\frac{\sqrt{121}}{2}=\frac{11-1}{2}=5$$

30. 다음 중 $\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$ 을 간단히 한 것은?

① $\frac{2}{13}$　② $\frac{4}{13}$　[✓ ③ $\frac{5}{14}$]　④ $\frac{23}{30}$　⑤ $\frac{31}{42}$

해설

$$\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$$

$$=\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+\frac{1}{6\times7}$$

$$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)$$

$$=\frac{1}{2}-\frac{1}{7}=\frac{5}{14}$$

31. $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{13\times 14}=\frac{a}{14}$ 에서 a의 값을 구하여라.

▶ 답 :

▷ 정답 : 13

해설

준식 $=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}\cdots-\frac{1}{14}=1-\frac{1}{14}=\frac{13}{14}$

$\therefore a=13$

32. $\dfrac{\sqrt{5}+1}{2} = 1 + \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}$ 을 만족하는 x 에 대하여 $[x]$ 의 값을 구하면?($[x]$ 는 x 를 넘지 않는 최대정수)

① 0 [✓ ② 1] ③ 2 ④ 3 ⑤ 4

해설

$$1 + \frac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}} = 1 + \frac{1}{1+\dfrac{x}{x+1}} = 1 + \frac{1}{\dfrac{2x+1}{x+1}}$$

$$= \frac{x+1}{2x+1} + 1 = \frac{x+1+2x+1}{2x+1}$$

$$= \frac{3x+2}{2x+1}$$

$$\frac{3x+2}{2x+1} = \frac{\sqrt{5}+1}{2}$$

$$(2\sqrt{5}+2-6)x = (3-\sqrt{5})$$

$$x = \frac{3-\sqrt{5}}{2\sqrt{5}-4} = \frac{(3-\sqrt{5})(2\sqrt{5}+4)}{(2\sqrt{5}-4)(2\sqrt{5}+4)}$$

$$= \frac{2(\sqrt{5}+1)}{4} = \frac{\sqrt{5}+1}{2}$$

$$2 < \sqrt{5} < 3\ ,\ 3 < \sqrt{5}+1 < 4$$

$$\frac{3}{2} < \frac{\sqrt{5}+1}{2} < 2$$

$$[x] = \left[\frac{\sqrt{5}+1}{2}\right] = 1$$

33. 다음 분수식 $\dfrac{x^2}{x-\dfrac{1}{x+\dfrac{1}{x}}}$ 을 간단히 하면?

① $\dfrac{x^2+1}{x^2}$ ② $\dfrac{x^2-1}{x^2}$ ③ $\dfrac{x^2+x+1}{x}$

[✓ ④ $\dfrac{x^2+1}{x}$] ⑤ $\dfrac{x^2-1}{x}$

해설

$$\frac{x^2}{x-\dfrac{1}{x+\dfrac{1}{x}}}=\frac{x^2}{x-\dfrac{x}{x^2+1}}=\frac{x^2}{\dfrac{x^3}{x^2+1}}=\frac{x^2+1}{x}$$

34. 유리식 $\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{a}}} \times \dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{a}}}$ 을 간단히 하면?

[✓ ① $1-a^2$]
② $(1-a)^2$
③ 1
④ $1+a^2$
⑤ $(1+a)^2$

해설

$$\frac{1}{1-\dfrac{1}{1-\dfrac{1}{a}}} = \frac{1}{1-\dfrac{a}{a-1}} = \frac{a-1}{a-1-a}$$

$$= 1-a$$

$$\frac{1}{1-\dfrac{1}{1+\dfrac{1}{a}}} = \frac{1}{1-\dfrac{a}{a+1}} = \frac{a+1}{a+1-a}$$

$$= 1+a$$

$\therefore$ (준 식) $= 1-a^2$

35. 번분수식 $1-\dfrac{\dfrac{1}{a}-\dfrac{2}{a+1}}{\dfrac{1}{a}-\dfrac{2}{a-1}}$ 를 간단히 하면?

① $\dfrac{a}{(a+1)^2}$ ② $\dfrac{2a}{(a+1)^2}$ ③ $\dfrac{3a}{(a+1)^2}$

[✓ ④] $\dfrac{4a}{(a+1)^2}$ ⑤ $\dfrac{5a}{(a+1)^2}$

해설

$$(\text{준 식}) = 1-\dfrac{\dfrac{-(a-1)}{a(a+1)}}{\dfrac{-(a+1)}{a(a-1)}} = 1-\dfrac{(a-1)^2}{(a+1)^2}$$

$$= \dfrac{4a}{(a+1)^2}$$

36. $2+\cfrac{1}{k+\cfrac{1}{m+\cfrac{1}{5}}}=\dfrac{803}{371}$일 때, 자연수 k, m의 값에 대하여 $k+m$의 값을 구하여라.

▶ 답 :

▷ 정답 : 18

해설

$$\frac{803}{371}=2+\frac{61}{371}=2+\cfrac{1}{\cfrac{371}{61}}$$

$$=2+\cfrac{1}{6+\cfrac{5}{61}}=2+\cfrac{1}{6+\cfrac{1}{\cfrac{61}{5}}}$$

$$=2+\cfrac{1}{6+\cfrac{1}{12+\cfrac{1}{5}}}$$

따라서 $k=6$, $m=12$

$\therefore k+m=18$

37. $2+\cfrac{1}{k+\cfrac{1}{m+\cfrac{1}{5}}}=\dfrac{803}{371}$ 일 때, 자연수 $k,\ m$의 값에 대하여 $k+m$의 값은?

① 6　② 12　[✓ ③ 18]　④ 24　⑤ 30

해설

$$\frac{803}{371}=2+\frac{61}{371}=2+\cfrac{1}{\cfrac{371}{61}}=2+\cfrac{1}{6+\cfrac{5}{61}}$$

$$=2+\cfrac{1}{6+\cfrac{1}{\cfrac{61}{5}}}=2+\cfrac{1}{6+\cfrac{1}{12+\cfrac{1}{5}}}$$

따라서 $k=6,\ m=12$

$\therefore\ k+m=18$

38. 등식 $\frac{225}{157} = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$ 을 만족시키는 자연수 $a,\ b,\ c,\ d,\ e$ 를 차례대로 구하여라.

▶ 답 :

▶ 답 :

▶ 답 :

▶ 답 :

▶ 답 :

▷ 정답 : $a = 1$

▷ 정답 : $b = 2$

▷ 정답 : $c = 3$

▷ 정답 : $d = 4$

▷ 정답 : $e = 5$

해설

$$\frac{225}{157} = 1 + \frac{68}{157} = 1 + \cfrac{1}{\cfrac{157}{68}}$$

$$= 1 + \cfrac{1}{2 + \cfrac{21}{68}} = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{5}{21}}}$$

$$= 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{5}}}}$$

$\therefore a = 1,\ b = 2,\ c = 3,\ d = 4,\ e = 5$

39. 유리수 $\frac{87}{19} = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e + \frac{1}{2}}}}}$ 로 나타낼 때, $a+b+c+d+e$ 의 값을 구하면?

① 7　② 8　[✓ ③ 9]　④ 10　⑤ 11

해설

$$\frac{87}{19} = 4 + \frac{11}{19} = 4 + \cfrac{1}{\frac{19}{11}}$$

$$= 4 + \cfrac{1}{1 + \frac{8}{11}}$$

$$\frac{8}{11} = \cfrac{1}{\frac{11}{8}} = \cfrac{1}{1 + \frac{3}{8}} = \cfrac{1}{1 + \cfrac{1}{\frac{8}{3}}}$$

$$= \cfrac{1}{1 + \cfrac{1}{2 + \frac{2}{3}}} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{2}}}}$$

$\therefore a = 4,\ b = 1,\ c = 1,\ d = 2,\ e = 1$

따라서 $a+b+c+d+e = 9$

40. 자연수 $a,\ b,\ c,\ d$에 대하여 $\dfrac{75}{23}=a+\dfrac{1}{b+\dfrac{1}{c+\dfrac{1}{d}}}$ 일 때, $a+b+c+d$의 값은?

① 8 ② 10 [✓ ③ 12] ④ 14 ⑤ 16

해설

$$\frac{75}{23}=3+\frac{6}{23}=3+\frac{1}{\frac{23}{6}}$$

$$=3+\frac{1}{3+\frac{5}{6}}=3+\frac{1}{3+\frac{1}{\frac{6}{5}}}$$

$$=3+\frac{1}{3+\frac{1}{1+\frac{1}{5}}}=a+\frac{1}{b+\frac{1}{c+\frac{1}{d}}}$$

$\therefore\ a=3,\ b=3,\ c=1,\ d=5$

$\therefore\ a+b+c+d=12$

41. $a+b+c=1$일 때, $\dfrac{a^2-1}{b+c}+\dfrac{b^2-1}{c+a}+\dfrac{c^2-1}{a+b}$의 값을 구하시오.

▶ 답 :

▷ 정답 : -4

해설

$$\frac{a^2-1}{b+c}+\frac{b^2-1}{c+a}+\frac{c^2-1}{a+b}$$

$$=\frac{(a-1)(a+1)}{b+c}+\frac{(b-1)(b+1)}{c+a}$$

$$+\frac{(c-1)(c+1)}{a+b}$$

그런데 $a+b+c=1$이므로

$a-1=-(b+c),\ b-1=-(c+a),\ c-1=-(a+b)$

$$\begin{aligned}\therefore(\text{준식}) &= -(a+1)-(b+1)-(c+1)\\ &= -(a+b+c)-3=-1-3=-4\end{aligned}$$

42. $\frac{3}{a}+\frac{3}{b}=\frac{16}{a+b}$ 일 때, $\frac{b}{a}+\frac{a}{b}$ 의 값을 구하면?

① $\frac{8}{3}$ [✓ ② $\frac{10}{3}$] ③ $\frac{14}{3}$ ④ $\frac{16}{3}$ ⑤ $\frac{17}{3}$

해설

$\frac{3}{a}+\frac{3}{b}=\frac{16}{a+b}$, $\frac{3(a+b)}{ab}=\frac{16}{a+b}$

$3(a+b)^2=16ab \quad \therefore\ 3a^2-10ab+3b^2=0$

$(3a-b)(a-3b)=0 \quad \therefore\ \frac{b}{a}=3,\ \frac{1}{3}$

$\therefore\ \frac{b}{a}+\frac{a}{b}=3+\frac{1}{3}=\frac{10}{3}$

43. $x + \frac{1}{x} = 2$일 때, $x^2 - \frac{1}{x^2}$의 값을 구하시오.

▶ 답 :

▷ 정답 : 0

해설

$$\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 = 2^2 - 4 = 0$$

$$\therefore x^2 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 0 \times 2 = 0$$

44. $x<0$이고 $x^4-x^2+1=0$일 때, $x+\frac{1}{x}$의 값을 구하면?

① $-\sqrt{2}$ [✓ ② $-\sqrt{3}$] ③ $-\sqrt{5}$ ④ $-\sqrt{6}$ ⑤ $-\sqrt{7}$

해설

$x^4-x^2+1=0$의 양변을 x^2으로 나누면

$x^2+\frac{1}{x^2}=1$

$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2=3$

$\therefore$ $x<0$이므로 $x+\frac{1}{x}=-\sqrt{3}$