1. $\sum_{k=1}^{5} a_k = 20$, $\sum_{k=1}^{5} b_k = 5$ 일 때, $\sum_{k=1}^{5}(2a_k - b_k - 1)$ 의 값은?

① 15 ② 20 ③ 25 ④ 30 ⑤ 35

해설

(주어진 식)

$= 2\sum_{k=1}^{5} a_k - \sum_{k=1}^{5} b_k - \sum_{k=1}^{5} 1$

$= 2 \cdot 20 - 5 - 5$

$= 30$

2. $\sum_{k=1}^{10} k^3$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 3025

해설

$\sum_{k=1}^{10} k^3 = \frac{10 \cdot 11}{2} \cdot \frac{10 \cdot 11}{2} = 3025$

3. $\sum_{l=1}^{10}\left\{\sum_{k=1}^{5}(k+l)\right\}$ 의 값은?

① 400　② 425　③ 450　④ 475　⑤ 500

해설

$\sum_{l=1}^{5}(k+l)=\sum_{k=1}^{5}k+\sum_{k=1}^{5}l=\sum_{k=1}^{5}k+5l$

$\therefore$ (준 식) $=\sum_{l=1}^{10}(5l+15)=5\sum_{l=1}^{10}l+150$

$=5\times 55+150=425$

4. 수열 $\frac{1}{1+\sqrt{2}}, \frac{1}{\sqrt{2}+\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{4}}, \cdots$ 의 제 15항까지의 합은?

① $\sqrt{14}-1$ ② $\sqrt{15}-1$ ③ 3
④ $\sqrt{15}+1$ ⑤ 5

해설

$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{15}+\sqrt{16}}$

$=\sum_{k=1}^{15}\frac{1}{\sqrt{k}+\sqrt{k+1}}$

$=\sum_{k=1}\frac{\sqrt{k}-\sqrt{k+1}}{(\sqrt{k}+\sqrt{k+1})(\sqrt{k}-\sqrt{k+1})}$

$=-\sum_{k=1}^{15}(\sqrt{k}-\sqrt{k+1})$

$=-\left\{(1-\sqrt{2})+(\sqrt{2}-\sqrt{3})+\cdots\right\}$

$-\left\{(\sqrt{15}-\sqrt{16})\right\}$

$=-(1-\sqrt{16})=\sqrt{16}-1=4-1=3$

5. $\sum_{k=1}^{n}\frac{1}{k^2+k}$ 의 값은?

① $\frac{1}{n+1}$ ② $\frac{n}{n+1}$ ③ $\frac{2n}{n+1}$
④ $\frac{2n}{2n+1}$ ⑤ $\frac{2n}{2n+3}$

해설

(주어진 식)$=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$
$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$
$=1-\frac{1}{n+1}=\frac{n}{n+1}$

6. 다음 등식이 성립하도록 하는 c의 값을 구하여라.

$$\sum_{k=11}^{100}(k-2)^2 = \sum_{k=11}^{100}k^2 - 4\sum_{k=11}^{100}k + c$$

▶ 답 :

▷ 정답 : 360

해설

$\sum_{k=11}^{100}(k-2)^2 = \sum_{k=11}^{100}(k^2-4k+4)$

$= \sum_{k=11}^{100} - 4\sum_{k=11}^{100}k + \sum_{k=11}^{100}4$

$\therefore\ c = \sum_{k=11}^{100}4 = 4+4+\cdots+4 = 4\times 90 = 360$

7. $1 \cdot 15 + 2 \cdot 14 + 3 \cdot 13 + \cdots + 15 \cdot 1$ 의 값은?

① 640 ② 660 ③ 680 ④ 700 ⑤ 720

해설

$n \le 15$ 일 때, $a_n = n(16-n) = -n^2 + 16n$

$\therefore 1 \cdot 15 + 2 \cdot 14 + 3 \cdot 13 + \cdots + 15 \cdot 1$

$= \sum_{k=1}^{15}(-k^2 + 16k) = -\sum_{k=1}^{15} k^2 + 16\sum_{k=1}^{15} k$

$= -\dfrac{15 \cdot 16 \cdot 31}{6} + 16 \cdot \dfrac{15 \cdot 16}{2} = 680$

8. 수열 $1 \cdot 2 \cdot 4,\ 2 \cdot 4 \cdot 8,\ 3 \cdot 6 \cdot 12,\ 4 \cdot 8 \cdot 16, \cdots$ 의 제 10 항까지의 합은?

① 400 ② 1100 ③ 12100
④ 24200 ⑤ 48400

해설

$a_k = k \cdot 2k \cdot 4k = 8k^3$ 이므로

$S_{10} = \sum_{k=1}^{10} 8k^3 = 8 \cdot \left(\frac{10 \cdot 11}{2}\right)^2 = 2 \cdot 10^2 \cdot 11^2 = 24200$

9. $\sum_{k=1}^{15} \log_2 \left(1+\frac{1}{k}\right)$ 의 값은?

① $\log_2 3$ ② $\log_2 15$ ③ $\log_2 30$
④ 3 ⑤ 4

해설

$\sum_{k=1}^{15} \log_2 \left(1+\frac{1}{k}\right) = \sum_{k=1}^{15} \log_2 \frac{k+1}{k}$
$= \sum_{k=1}^{15} \{\log_2(k+1) - \log_2 k\}$
$= (\log_2 2 - \log_2 1) + (\log_2 3 - \log_2 2) + \cdots$
$+ (\log_2 16 - \log_2 15)$
$= \log_2 16 - \log_2 1 = \log_2 2^4 = 4$

10. 첫째항부터 제 n항까지의 합 S_n이 $S_n = 2n^2 - n + 3$인 수열 $\{a_n\}$에서 $\sum_{k=1}^{5} a_{2k-1}$ 의 값은?

① 82　② 84　③ 86　④ 88　⑤ 90

해설

$S_n = 2n^2 - n + 3$이므로

$$\begin{aligned} a_n &= S_n - S_{n-1} \\ &= 2n^2 - n + 3 - \{2(n-1)^2 - (n-1) + 3\} \\ &= 4n - 3 \ (n \ge 2) \end{aligned}$$

$a_1 = S_1 = 2 - 1 + 3 = 4$

$$\begin{aligned} \therefore \sum_{k=1}^{5} a_{2k-1} &= a_1 + a_3 + a_5 + a_7 + a_9 \\ &= 4 + 9 + 17 + 25 + 33 = 88 \end{aligned}$$

11. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$ 의 값은?

① $\frac{n(3n+5)}{4(n+1)(n+2)}$ ② $\frac{n(3n+5)}{4(2n+1)(n+2)}$

③ $\frac{n(3n+5)}{(n+1)(n+2)}$ ④ $\frac{n(3n+4)}{4(n+1)(n+2)}$

⑤ $\frac{n(3n+4)}{2(n+1)(n+2)}$

해설

$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$

$=\sum_{k=1}^{n}\frac{1}{k(k+2)}$

$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\}$

$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$

$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\frac{n(3n+5)}{4(n+1)(n+2)}$

12. 함수 $f(n)=1^2+2^2+3^2+\cdots+n^2$에 대하여 $\sum_{k=1}^{20}\frac{2k+1}{f(k)}$의 값은?

① $\frac{40}{7}$ ② $\frac{45}{8}$ ③ $\frac{17}{3}$ ④ $\frac{57}{10}$ ⑤ $\frac{63}{11}$

해설

$f(n)=1^2+2^2+3^2+\cdots+n^2$

$=\sum_{k=1}^{20}k^2=\frac{n(n+1)(2n+1)}{6}$이므로

$\sum_{k=1}^{20}\frac{2k+1}{f(x)}=\sum_{k=1}^{20}\frac{2k+1}{\frac{k(k+1)(2k+1)}{6}}$

$=\sum_{k=1}^{20}\frac{6}{k(k+1)}=6\sum_{k=1}^{20}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=6\left(1-\frac{1}{21}\right)=6\times\frac{20}{21}=\frac{40}{7}$

13. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+10}$ 의 값은?

① $\frac{9}{10}$ ② $\frac{11}{10}$ ③ $\frac{10}{11}$ ④ $\frac{20}{11}$ ⑤ $\frac{11}{20}$

해설

$\frac{1}{1+2+\cdots+n}=\frac{1}{\frac{n(n+1)}{2}}=\frac{2}{n(n+1)}$

$\therefore \sum_{k=1}^{10}\frac{2}{k(k+1)}=2\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=2\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right)\right\}$

$=2\left(1-\frac{1}{11}\right)=\frac{20}{11}$

14. 수열의 합 $S = 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}$을 간단히 하면? (단, $x \neq 1$)

① $S = \dfrac{n(1-x^n)}{2}$　　② $S = \dfrac{1-x^n}{2}$

③ $S = \dfrac{1-x^n}{2} - \dfrac{2x^n}{x}$　　④ $S = \dfrac{1-x^n}{1+x} - \dfrac{1-x^n}{(1-x)^2}$

⑤ $S = \dfrac{1-x^n}{(1-x)^2} - \dfrac{nx^n}{1-x}$

해설

등차수열과 등비수열의 곱으로 이루어진 멱급수의 형태이므로 양변에 x를 곱하여 변끼리 빼면

$$\begin{array}{rl} S = & 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} \\ -)\ xS = & \quad\ x + 2x^2 + 3x^3 + \cdots + (n-1)x^n + nx^n \\ \hline (1-x)S = & 1 + x + x^2 + x^3 + \cdots + x^n - n \cdot x \end{array}$$

$= \dfrac{1(1-x^n)}{1-x} - n \cdot x^n$

$\therefore S = \dfrac{1-x^n}{(1-x)^2} - \dfrac{nx^n}{1-x}$

15. 다음 수열의 합을 구하여라.

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9$$

▶ 답 :

▷ 정답 : 8194

해설

$S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9 \cdots$ ㉠

$2S = 1 \cdot 2^2 + 2 \cdot 2^3 + \cdots + 8 \cdot 2^9 + 9 \cdot 2^{10} \cdots$ ㉡

이므로 ㉠-㉡을 하면

$$\begin{aligned} -S &= \frac{2(2^9 - 1)}{2 - 1} - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 2 - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 18 \cdot 2^9 - 2 \\ &= -16 \cdot 2^9 - 2 \end{aligned}$$

$\therefore S = 2^{13} + 2 = 1024 \times 8 + 2 = 8194$

16. $\sum_{k=1}^{10}\left\{\sum_{m=1}^{n}(k-2)\cdot 2^{m-1}\right\}$을 n에 관한 식으로 나타내면?

① $60(2^n-1)$ ② $35(2^n-1)$ ③ $20(2^n+1)$
④ $20(2^n-1)$ ⑤ $16(2^n-1)$

해설

$\sum_{k=1}^{10}\left\{\sum_{m=1}^{n}(k-2)\cdot 2^{m-1}\right\}$

$=\sum_{k=1}^{10}\left\{\frac{(k-2)(2^n-1)}{2-1}\right\}$

$=(2^n-1)\sum_{k=1}^{10}(k-2)$

$=(2^n-1)\left(\frac{10\times 11}{2}-20\right)=35(2^n-1)$

17. $a_1 + a_3 + a_5 + \cdots + a_{99}$ 를 $\sum$ 를 이용하여 나타내면?

① $\sum_{k=1}^{99} a_k$ ② $\sum_{k=1}^{99} a_{2k-1}$ ③ $\sum_{k=1}^{99} a_{2k+1}$

④ $\sum_{k=1}^{50} a_k$ ⑤ $\sum_{k=1}^{50} a_{2k-1}$

해설

① $\sum_{k=1}^{99} a_k = a_1 + a_2 + a_3 + \cdots + a_{99}$

② $\sum_{k=1}^{99} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{197}$

③ $\sum_{k=1}^{99} a_{2k+1} = a_3 + a_5 + a_7 + \cdots + a_{199}$

④ $\sum_{k=1}^{50} a_k = a_1 + a_2 + a_3 + \cdots + a_{50}$

⑤ $\sum_{k=1}^{50} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{99}$

18. $\sum_{k=1}^{5}(2k-1)+\sum_{k=6}^{10}(2k-1)$ 의 값은?

① 70 ② 80 ③ 90 ④ 100 ⑤ 110

해설

$\sum_{k=1}^{5}(2k-1)+\sum_{k=6}^{10}(2k-1)$
$=\sum_{k=1}^{10}(2k-1)=2\cdot\sum_{k=1}^{10}k-\sum_{k=1}^{10}1$
$=2\cdot\frac{10\cdot 11}{2}-10$
$=110-10=100$

19. $4^3+5^3+6^3+\cdots+10^3$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 2989

해설

$4^3+5^3+6^3+\cdots+10^3=\sum_{k=1}^{10}k^3-\sum_{k=1}^{3}k^3$

$=\left(\frac{10\cdot 11}{2}\right)^2-\left(\frac{3\cdot 4}{2}\right)^2$

$=3025-36=2989$

20. 다음 중 옳은 것은?

① $1+4+7+\cdots+(3n-5)=\sum_{k=1}^{n}(3k-5)$

② $2+4+6+\cdots+2(n+1)=\sum_{k=1}^{n}2(k+1)$

③ $3+5+7+\cdots+(2n-1)=\sum_{k=1}^{n}(2k+1)$

④ $4+5+6+\cdots+(n+3)=\sum_{k=1}^{n}(k+3)$

⑤ $3+4+5+\cdots+n=\sum_{k=1}^{n}k$

해설

① $1+4+7+\cdots+(3n-5)=\sum_{k=1}^{n-1}(3k-2)$
② $2+4+6+\cdots+2(n+1)=\sum_{k=1}^{n+1}2n$
③ $3+5+7+\cdots+(2n-1)=\sum_{k=1}^{n-1}(2k+1)$
⑤ $3+4+5+\cdots+n=\sum_{k=1}^{n-2}(k+2)$