1. $a_1 + a_3 + a_5 + \cdots + a_{99}$를 $\sum$를 이용하여 나타내면?

① $\sum_{k=1}^{99} a_k$ ② $\sum_{k=1}^{99} a_{2k-1}$ ③ $\sum_{k=1}^{99} a_{2k+1}$

④ $\sum_{k=1}^{50} a_k$ ⑤ $\sum_{k=1}^{50} a_{2k-1}$

해설

① $\sum_{k=1}^{99} a_k = a_1 + a_2 + a_3 + \cdots + a_{99}$
② $\sum_{k=1}^{99} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{197}$
③ $\sum_{k=1}^{99} a_{2k+1} = a_3 + a_5 + a_7 + \cdots + a_{199}$
④ $\sum_{k=1}^{50} a_k = a_1 + a_2 + a_3 + \cdots + a_{50}$
⑤ $\sum_{k=1}^{50} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{99}$

2. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$ 의 값은?

① 385　　② 550　　③ 1100　　④ 1150　　⑤ 1200

해설

$$\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$$

$$=\sum_{j=1}^{10}\left\{3j+\frac{j(j+1)}{2}\right\}$$

$$=\sum_{j=1}^{10}\left(\frac{j^2+7i}{2}\right)$$

$$=\frac{1}{2}\left(\sum_{j=1}^{10}j^2+7\sum_{j=1}^{10}j\right)$$

$$=\frac{1}{2}\left(\frac{10\cdot 11\cdot 21}{6}+7\times\frac{10\cdot 11}{2}\right)$$

$$=\frac{1}{2}(385+385)=385$$

3. $\sum_{k=1}^{n} a_k = 10n$, $\sum_{k=1}^{n} b_k = 5n$ 일 때, $\sum_{n=1}^{10} \left\{\sum_{k=1}^{n}(2a_k - 3b_k + 5)\right\}$ 의 값은?

① 250 ② 300 ③ 450 ④ 550 ⑤ 650

해설

$$\sum_{n=1}^{10} \left\{2\sum_{k=1}^{n} a_k - 3\sum_{k=1}^{n} b_k + \sum_{k=1}^{n} 5\right\}$$
$$= \sum_{n=1}^{10}(2 \cdot 10n - 3 \cdot 5n + 5n)$$
$$= \sum_{n=1}^{10}(20n - 15n + 5n)$$
$$= \sum_{n=1}^{10} 10n = 10 \cdot \frac{10 \cdot 11}{2}$$
$$= 550$$

4. 다음 수열의 합을 $\sum$ 기호를 써서 나타내면?

$3 + 6 + 12 + \cdots + 3 \cdot 2^{n-1}$

① $\sum_{k=1}^{n} 3 \cdot 2^{k-1}$　② $\sum_{k=1}^{n-1} 3 \cdot 2^{k-1}$　③ $\sum_{k=1}^{n} 3 \cdot 2^{k}$

④ $\sum_{k=1}^{n-1} 3 \cdot 2^{k}$　⑤ $\sum_{k=1}^{n} 3 \cdot 2^{k+1}$

해설

제 k항은 $3 \cdot 2^{k-1}$, 항 수는 n이므로

$3 + 6 + 9 + \cdots + 3 \cdot 2^{n-1} = \sum_{k=1}^{n} 3 \cdot 2^{k-1}$

5. $\sum_{k=1}^{10} \log \dfrac{k+2}{k}$ 의 값은?

① $\log 45$ ② $\log 50$ ③ $\log 55$ ④ $\log 60$ ⑤ $\log 66$

해설

$$\sum_{k=1}^{10} \log \frac{k+2}{k}$$

$$= \log \frac{3}{1} + \log \frac{4}{2} + \log \frac{5}{3} + \cdots + \log \frac{11}{9} + \log \frac{12}{10}$$

$$= \log \left(\frac{3}{1} \cdot \frac{4}{2} \cdot \frac{5}{3} \cdots \frac{11}{9} \cdot \frac{12}{10} \right)$$

$$= \log \frac{11 \cdot 12}{1 \cdot 2} = \log 66$$

6. 다음 중 $\sum_{k=1}^{10} k + \sum_{k=2}^{10} k + \cdots + \sum_{k=10}^{10} k$의 값과 같은 것은?

① $\sum_{k=1}^{10} 2k$ ② $\sum_{k=1}^{20} k$ ③ $\sum_{k=6}^{10} 5k$

④ $\sum_{k=1}^{10} k^2$ ⑤ $\sum_{k=1}^{10} (k^2 + k)$

해설

$\sum_{k=1}^{10} k + \sum_{k=2}^{10} k + \cdots + \sum_{k=10}^{10} k$
$= (1 + 2 + 3 + 4 + \cdots + 10) + (2 + 3 + 4 + \cdots + 10) + (3 + 4 + \cdots + 10) + \cdots + 10$
$= 1 + 2 \cdot 2 + 3 \cdot 3 + \cdots + 10 \cdot 10 = \sum_{k=1}^{10} k^2$

7. 다음 $\sum$의 성질 중 옳지 않은 것은?

① $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

② $\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

③ $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$(단, c는 상수)

④ $\sum_{k=1}^{n} c = cn$(단, c는 상수)

⑤ $\sum_{k=1}^{n}(a_k + c) = \sum_{k=1}^{n} a_k + c$(단, c는 상수)

해설

$\sum_{k=1}^{n}(a_k + c) = \sum_{k=1}^{n} a_k + cn$

8. $\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1)$을 n에 대한 식으로 나타내면 an^2+bn+c일 때, 상수 a, b, c의 곱 abc의 값은?

① -2 ② -1 ③ 0 ④ 1 ⑤ 2

해설

$$\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1)$$
$$=\sum_{k=1}^{n-1}(k^2+1)+(n^2+1)-\sum_{k=1}^{n-1}(k^2-1)$$
$$=\sum_{k=1}^{n-1}\{k^2+1-(k^2-1)\}+(n^2+1)$$
$$=\sum_{k=1}^{n-1}2+(n^2+1)$$
$$=2(n-1)+(n^2+1)=n^2+2n-1$$
$$\therefore a=1,\ b=2,\ c=-1$$
$$\therefore abc=-2$$

9. 두 수열 a_n, b_n에 대하여 $a_n = n^3 + 3n^2 + 2n$, $b_n = n^2 + n$일 때, $\sum_{i=1}^{4}\left(\sum_{j=1}^{3} a_i b_j\right)$의 값은?

① 4000 ② 4100 ③ 4200 ④ 4300 ⑤ 4400

해설

$a_n = n^3 + 3n^2 + 2n = n(n+1)(n+2)$

$b_n = n^2 + n = n(n+1)$

$\therefore\ \sum_{i=1}^{4}\left(\sum_{j=1}^{3} a_i b_j\right) = \sum_{i=1}^{4} a_i\left(\sum_{j=1}^{3} b_j\right)$

$= \left(\sum_{i=1}^{4} a_i\right) \times \left(\sum_{j=1}^{3} b_j\right)$

$= \left\{\sum_{i=1}^{4} i(i+1)(i+2)\right\} \times \sum_{j=1}^{3} j(j+1)$

$= \sum_{i=1}^{4}(i^3 + 3i^2 + 2i) \times \sum_{j=1}^{3}(j^2 + j)$

$$= \left\{\left(\frac{4\cdot 5}{2}\right)^2 + 3\cdot\frac{4\cdot 5\cdot 9}{6} + 2\cdot\frac{4\cdot 5}{2}\right\}$$

$$\times\left(\frac{3\cdot 4\cdot 7}{6} + \frac{3\cdot 4}{2}\right)$$

$= 210 \times 20 = 4200$

10. $\sum_{l=1}^{n}\left(\sum_{k=1}^{l} k\right)=56$ 을 만족시키는 n 의 값은?

① 5 ② 6 ③ 7 ④ 8 ⑤ 9

해설

$\sum_{l=1}^{n}\left(\sum_{k=1}^{l} k\right)$

$=\sum_{l=1}^{n}\left\{\frac{l(l+1)}{2}\right\}=\frac{1}{2}\left(\sum_{l=1}^{n} l^2+\sum_{l=1}^{n} l\right)$

$=\frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right\}$

$=\frac{n(n+1)(n+2)}{6}$

즉, $\frac{n(n+1)(n+2)}{6}=56$ 이므로

$n(n+1)(n+2)=6\cdot 7\cdot 8$

$\therefore n=6$

11. n개의 수 $1 \cdot 2n,\ 2 \cdot (2n-1),\ 3 \cdot (2n-2), \cdots,\ n(n+1)$ 의 합은?

① $\dfrac{n^2(n+1)}{2}$ ② $\dfrac{n(n+1)^2}{2}$

③ $\dfrac{(n+1)(2n+1)}{6}$ ④ $\dfrac{(n+1)(2n+1)}{3}$

⑤ $n(n+1)(2n+1)$

해설

주어진 수열의 제 k항은

$k\left\{2n-(k-1)\right\} = k(2n-k+1)$

$= -k^2 + (2n+1)k$

이므로 구하는 합은

$\sum_{k=1}^{n} k\left\{2n-(k-1)\right\}$

$= -\sum_{k=1}^{n} k^2 + (2n+1)\sum_{k=1}^{n} k$

$= -\dfrac{n(n+1)(2n+1)}{6} + (2n+1) \times \dfrac{n(n+1)}{2}$

$= \dfrac{n(n+1)(2n+1)}{3}$

12. $1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1$ 의 값은?

① 1310 ② 1320 ③ 1330 ④ 1340 ⑤ 1350

해설

$$1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1$$
$$= 1 \cdot (20 - 1) + 2 \cdot (20 - 2) + 3 \cdot (20 - 3) + \cdots + 19 \cdot (20 - 19)$$
$$= \textstyle\sum_{k=1}^{19} k(20 - k) = \sum_{k=1}^{19} (20k - k^2)$$
$$= 20 \times \frac{19 \cdot 20}{2} - \frac{19 \cdot 20 \cdot 39}{6}$$
$$= 190(20 - 13) = 1330$$

13. 방정식 $x^3-1=0$의 두 허근을 α, β라고 할 때, $\sum_{k=1}^{3}(\alpha^k+\beta^k)$의 값은?

① -4 ② -3 ③ -2 ④ -1 ⑤ 0

해설

$x^3-1=(x-1)(x^2+x+1)=0$에서 두 허근 α, β는
$x^2+x+1=0$의 근이므로
$\alpha+\beta=-1,\ \alpha\beta=1,\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=-1$
$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=2$
$\therefore\ \sum_{k=1}^{3}(\alpha^k+\beta^k)=(\alpha+\beta)+(\alpha^2+\beta^2)+(\alpha^3+\beta^3)$
$=(-1)+(-1)+2=0$

14. 이차방정식 $x^2-2x-5=0$의 두 근을 α, β라 할 때, $\sum_{k=1}^{10}(\alpha-k)(\beta-k)$의 값은?

① 215 ② 225 ③ 235 ④ 245 ⑤ 255

해설

이차방정식의 근과 계수의 관계에 의하여

$\alpha+\beta=2,\ \alpha\beta=-5$

$\therefore \sum_{k=1}^{10}(\alpha-k)(\beta-k)$

$=\sum_{k=1}^{10}\left\{k^2-(\alpha+\beta)k+\alpha\beta\right\}$

$=\sum_{k=1}^{10}(k^2-2k-5)$

$=\dfrac{10\cdot 11\cdot 21}{6}-2\times\dfrac{10\cdot 11}{2}-50=225$

15. $x_i \in \{0,\ 1,\ 2\}$이고, $\sum_{i=1}^{n} x_i = 20$, $\sum_{i=1}^{n} x_i^2 = 34$일 때, $\sum_{i=1}^{n} x_i^3$의 값은?

① 62 ② 74 ③ 86 ④ 98 ⑤ 110

해설

x_i 중 1을 a개, 2를 b개 택한다면

$\sum_{i=1}^{n} x_i = 1 \times a + 2 \times b = 20 \quad \therefore a + 2b = 20 \cdots\cdots$ ㉠

$\sum_{i=1}^{n} x_i^2 = 1^2 \times a + 2^2 \times b = 34 \quad \therefore a + 4b = 34 \cdots\cdots$ ㉡

㉠, ㉡에서 $a = 6,\ b = 7$

$\therefore \sum_{i=1}^{n} x_i^3 = 1^3 \times 6 + 2^3 \times 7 = 6 + 56 = 62$

16. 등비수열 $\{a_n\}$에 대하여 $A = \sum_{k=1}^{10} a_{2k-1}$, $B = \sum_{k=1}^{10} a_{2k}$라 할 때, 다음 중 이 수열의 공비 r을 나타내는 것은?(단, $a_1 \neq 0$, $r > 0$)

① $\dfrac{B}{A}$ ② $\dfrac{A}{B}$ ③ $\sqrt{\dfrac{B}{A}}$ ④ $\sqrt{\dfrac{A}{B}}$ ⑤ $\sqrt{AB}$

해설

$A = \sum_{k=1}^{10} a_{2k-1} = a_1 + a_3 + a_5 + \cdots a_{19}$

$= a + ar^2 + ar^4 + \cdots + ar^{18}$

$B = \sum_{k=1}^{10} = a_2 + a_4 + a_6 + \cdots + a_{20}$

$= ar + ar^3 + ar^5 + \cdots + ar^{19}$

$= r\left\{a + ar^2 + ar^4 + \cdots + ar^{18}\right\} = r \cdot A$

따라서 $r = \dfrac{B}{A}$

17. 수열 $\{a_n\}$이 $\sum_{k=1}^{n} a_{2k-1} = n^2$, $\sum_{k=1}^{n} a_{2k} = 2^n$을 만족할 때, $a_9 + a_{10}$의 값은?

① 20　② 22　③ 25　④ 27　⑤ 30

해설

$n \geq 2$일 때,

$a_{2n-1} = \sum_{k=1}^{n} a_{2k-1} - \sum_{k=1}^{n-1} a_{2k-1} = n^2 - (n-1)^2 = 2n - 1$

$\therefore\ a_9 = 2 \cdot 5 - 1 = 9$

$a_{2n} = \sum_{k=1}^{n} a_{2k} - \sum_{k=1}^{n-1} a_{2k} = 2^n - 2^{n-1} = 2^{n-1}$

$\therefore\ a_{10} = 2^{5-1} = 16$

$\therefore\ a_9 + a_{10} = 25$

18. 수열$\sum_{k=1}^{8}(2k-1)\cdot 2^{k-1}$ 의 합을 구하여라.

▶ 답 :

▷ 정답 : 3331

해설

$S = 1\cdot 1 + 3\cdot 2 + 5\cdot 2^2 + \cdots + 13\cdot 2^6 + 15\cdot 2^7 \cdots\cdots$ ㉠
$2S = 1\cdot 2 + 3\cdot 2^2 + 5\cdot 2^3 + \cdots + 13\cdot 2^7 + 15\cdot 2^8 \cdots\cdots$ ㉡
이므로 ㉠ - ㉡을 하면

$$-S = 2\cdot\frac{(2^8-1)}{2-1} - 1 - 15\cdot 2^8$$

$$S = -2\cdot 2^8 + 2 + 1 + 15\cdot 2^8$$
$$= 13\cdot 2^8 + 3 = 3331$$

19. 첫째항이 0이고 공차가 0이 아닌 등차수열 $\{a_n\}$에 대하여 수열 $\{b_n\}$이 $a_{n+1}b_n = \sum_{k=1}^{n} a_k$를 만족시킬 때, b_{27}의 값을 구하여라.

▶ 답 :

▷ 정답 : 13

해설

등차수열 $\{a_n\}$의 공차를 $d(d \neq 0)$라 하면

$a_n = (n-1)d$

$a_{n+1}b_n = \sum_{k=1}^{n} a_k$에서 $nd \cdot b_n = \sum_{k=1}^{n}(k-1)d$

$nd \cdot b_n = d\left\{\frac{n(n+1)}{2} - n\right\}$, $b_n = \frac{n+1}{2} - 1 = \frac{n-1}{2}$

$b_{27} = \frac{27-1}{2} = 13$

20. 모든 자연수 n에 대하여 수열 $\{a_n\}$은 다음을 만족시킨다. 이때, a_{10}의 값을 구하여라.

$$\sum_{k=1}^{n} \frac{a_1 + \cdots + a_k}{k} = (n+1)^2$$

▶ 답 :

▷ 정답 : 39

해설

$\dfrac{a_1 + \cdots + a_n}{k} = b_n$ 이라 하면 $\sum_{k=1}^{n} b_k = (n+1)^2$

$$\begin{cases} b_n = (n+1)^2 - n^2 = 2n+1 \ (n \geq 2) \\ b_1 = 4 \end{cases}$$

$$\begin{cases} a_1 + \cdots + a_n = n \cdot (2n+1) = 2n^2 + n \ (n \geq 2) \\ a_1 = b_1 = 4 \end{cases}$$

$$\begin{aligned} a_{10} &= 2 \cdot 10^2 + 10 - (2 \cdot 9^2 + 9) \\ &= 39 \end{aligned}$$