1. $\log\left(1-\frac{1}{4}\right)+\log\left(1-\frac{1}{9}\right)+\log\left(1-\frac{1}{16}\right)+\cdots+\log\left(1-\frac{1}{64}\right)$ 을 간단히 하면?

[✓ ① $2\log 3-4\log 2$]
② $3\log 2-2\log 3$
③ $3\log 3-4\log 2$
④ $4\log 2-3\log 3$
⑤ $4\log 3-2\log 2$

해설

$1-\frac{1}{n^2}=\frac{(n-1)(n+1)}{n^2}$ 이므로

(주어진 식) $= \log \frac{1\cdot 3}{2^2}\times\frac{2\cdot 4}{3^2}\times\cdots\times\frac{7\cdot 9}{8^2}$

$= \log \frac{1}{2}\times\frac{9}{8}=\log\frac{9}{16}$

$= \log 9-\log 16$

$= 2\log 3-4\log 2$

2. $\log_2 6 - \log_2 \frac{3}{2}$ 의 값을 구하면?

① 0 ② −1 ③ 1 ④ −2 [✓ ⑤ 2]

해설

로그의 성질에 의하여
$\log_a x - \log_a y = \log_a \frac{x}{y}$ 이므로

$$\begin{aligned}\log_2 6 - \log_2 \frac{3}{2} &= \log_2\left(6 \div \frac{3}{2}\right)\\ &= \log_2\left(6 \times \frac{2}{3}\right) = 2\end{aligned}$$

3. $1 + \log_9 12 - \log_9 4$의 값은?

① $\frac{1}{2}$　② 1　[✓ ③ $\frac{3}{2}$]　④ 2　⑤ $\frac{5}{2}$

해설

$$\begin{aligned}\log_9 12 - \log_9 4 &= 1 + \log_9 9 + \log_9 12 - \log_9 4 \\ &= \log_9 (9 \times 12 \div 4) \\ &= \log_9 27 = \log_{3^2} 3^3 = \frac{3}{2}\end{aligned}$$

4. $\log_2 5\sqrt{3} + \log_2 \frac{24}{5} - \log_2 3\sqrt{3}$ 의 값은?

① 2 [✓ ② 3] ③ 5 ④ $\log_2 5$ ⑤ $\log_2 6$

해설

$$\log_2 5\sqrt{3} + \log_2 \frac{24}{5} - \log_2 3\sqrt{3} = \log_2 \frac{5\sqrt{3} \times \frac{24}{5}}{3\sqrt{3}}$$

$$= \log_2 8 = \log_2 2^3 = 3$$

5. $(\log_3 2)(\log_4 9) - \log_4 36$ 의 값은?

[✓ ① $-\log_2 3$]　② $-\log_3 2$　③ 0

④ $\log_3 2$　⑤ $\log_2 3$

해설

$(\log_3 2)(\log_4 9) - \log_4 36$

$= (\log_3 2)(\log_2 3) - \log_2 6$

$= 1 - \log_2 6 = -\log_2 3$

6. $(\log_3 2)(\log_4 25) - \log_9 75$ 의 값은?

[✓ ① $-\frac{1}{2}$]　② -1　③ 0　④ $\log_3 2$　⑤ $\log_2 3$

해설

$$(\log_3 2)(\log_4 25) - \log_9 75$$
$$= (\log_3 2)(\log_2 5) - \log_9 75$$
$$= \log_3 5 - \frac{1}{2}\log_3 75$$
$$= \log_3 \frac{5}{5\sqrt{3}}$$
$$= \log_3 \frac{1}{\sqrt{3}}$$
$$= -\frac{1}{2}$$

7. $\log_4 2 + \log_8 4 - \log_{16} 8$ 의 값은?

① $-\frac{1}{12}$ ② $-\frac{1}{2}$ ③ $\frac{1}{12}$ ④ 1 [✓ ⑤ $\frac{5}{12}$]

해설

$\log_{2^2} 2 + \log_{2^3} 2^2 - \log_{2^4} 2^3$

$= \frac{1}{2} + \frac{2}{3} - \frac{3}{4} = \frac{6+8-9}{12}$

$= \frac{5}{12}$

8. $3^{\log_4 5^{\log_3 4}}$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 5

해설

$3^{\log_4 5^{\log_3 4}} = 3^{\log_3 4 \cdot \log_4 5} = 3^{\log_3 5} = 5$

9. $\log_3 \sqrt{6} - \frac{1}{2}\log_3 \frac{1}{5} - \frac{3}{2}\log_3 \sqrt[3]{30}$ 을 계산하면?

[✓ ① 0]
② $\frac{1}{2}$
③ $-\frac{1}{2}$
④ $\frac{1}{2}\log_3 2$
⑤ $-\frac{1}{2}\log_3 2$

해설

$\log_3 \sqrt{6} - \frac{1}{2}\log_3 \frac{1}{5} - \frac{3}{2}\log_3 \sqrt[3]{30}$

$= \frac{1}{2}\log_3 6 + \frac{1}{2}\log_3 5 - \frac{3}{2} \cdot \frac{1}{3}\log_3 30$

$= \frac{1}{2}(\log_3 6 + \log_3 5 - \log_3 30)$

$\therefore \frac{1}{2}(\log_3 30 - \log_3 30) = 0$

10. $2\log_3 \frac{2}{3} + \log_3 \sqrt{72} - \frac{1}{2}\log_3 8$을 간단히 하면?

① $\log_3 2$
② $\log_3 2 - 1$
[✓ ③ $2\log_3 2 - 1$]
④ $\log_3 +1$
⑤ $2\log_3 2$

해설

$$2\log_3 \frac{2}{3} + \log_3 \sqrt{72} - \frac{1}{2}\log_3 8$$
$$= \log_3 \left(\frac{2}{3}\right)^2 + \log_3 6\sqrt{2} - \log_3 \sqrt{8}$$
$$= \log_3 \frac{4}{9} + \log_3 6\sqrt{2} - \log_3 2\sqrt{2}$$
$$= \log_3 \left(\frac{4}{9} \times 6\sqrt{2} \times \frac{1}{2\sqrt{2}}\right)$$
$$= \log_3 \frac{4}{3}$$
$$= 2\log_3 2 - 1$$

11. $\frac{1}{2}\log_2 3 + 5\log_2 \sqrt{2} - \log_2 \sqrt{6}$의 값은?

① 0　　② 1　　③ $\frac{3}{2}$　　[✓ ④ 2]　　⑤ $\frac{5}{2}$

해설

$\frac{1}{2}\log_2 3 + 5\log_2 \sqrt{2} - \log_2 \sqrt{6}$

$= \log_2 \sqrt{3} + \log_2 4\sqrt{2} - \log_2 \sqrt{6}$

$= \log_2 \frac{\sqrt{3} \times 4\sqrt{2}}{\sqrt{6}}$

$= \log_2 4$

$= 2$

12. $x = \sqrt{11} + \sqrt{3}, y = \sqrt{11} - \sqrt{3}$ 일 때, $\log_6(x^2 + xy + y^2)$ 의 값은?

① $\log_6 25$ [✓ ② 2] ③ 3

④ $\log_2 12$ ⑤ 6

해설

$x = \sqrt{11} + \sqrt{3},\ y = \sqrt{11} - \sqrt{3}$ 이므로
$(x+y)^2 = (2\sqrt{11})^2 = 44$
$xy = (\sqrt{11} + \sqrt{3})(\sqrt{11} - \sqrt{3})$
$= 11 - 3 = 8$
$\therefore\ x^2 + xy + y^2 = (x+y)^2 - xy = 44 - 8 = 36$
$\therefore\ \log_6(x^2 + xy + y^2) = \log_6 36 = \log_6 6^2 = 2\log_6 6 = 2$

13. $\log_{10} 2 = a,\ \log_{10} 3 = b$ 일 때, $\log_{10} 12$를 $a,\ b$로 나타내면?

① $2ab$　② a^2b　[✓ ③ $2a + b$]

④ $a^2 + b$　⑤ $a + 2b$

해설

$\log_{10} 12 = \log_{10}(3 \times 2^2)$

$= \log_{10} 3 + \log_{10} 2^2$

$= \log_{10} 3 + 2\log_{10} 2$

$= b + 2a$

14. $\log_3 2 = a,\ \log_3 5 = b$라고 할 때, $\log_8 125$를 $a,\ b$로 나타내면?

① $1-2b$ ② $2b-a$ ③ $a-b$
[✓ ④ $\frac{b}{a}$] ⑤ $\frac{a}{b}$

해설

$\log_3 2 = a \quad \log_3 5 = b$
$\log_8 125 = \log_{2^3} 5^3 = \log_2 5$
$= \frac{\log_3 5}{\log_3 2} = \frac{b}{a}$

15. $3^x = 2 + \sqrt{2}$, $3^y = 2 - \sqrt{2}$일 때, $x + y$의 값은?

① 1
② $\log_4 3$
[✓ ③ $\log_3 2$]
④ $\log_3 4$
⑤ $\log_4 10$

해설

$x = \log_3(2 + \sqrt{2})$, $y = \log_3(2 - \sqrt{2})$ 이므로

$x + y = \log_3 \left\{(2 + \sqrt{2})(2 - \sqrt{2})\right\} = \log_3 2$