

1. -8 의 세제곱근 중에서 실수를 a , 16 의 네제곱근 중에서 실수를 b 라 할 때, $a + b^2$ 의 값을 구하면?

① 0

② 2

③ 4

④ 6

⑤ 8

해설

$$a = -2$$

$$b = 2 \text{ or } -2$$

$$a + b^2 = -2 + 4 = 2$$

2. $\sqrt[3]{9^4} \div \sqrt{3^3} \times \sqrt[6]{\frac{1}{3}}$ 의 값을 구하면?

① 9

② 3

③ $\sqrt{3}$

④ $\frac{\sqrt{3}}{3}$

⑤ $\frac{1}{3}$

해설

$$\begin{aligned}\sqrt[3]{9^4} \div \sqrt{3^3} \times \sqrt[6]{\frac{1}{3}} &= (3^2)^{\frac{4}{3}} \div 3^{\frac{3}{2}} \times (3^{-1})^{\frac{1}{6}} \\ &= 3^{\frac{8}{3} - \frac{3}{2} - \frac{1}{6}} \\ &= 3\end{aligned}$$

3. 양수 a 에 대하여 $(a^{2\sqrt{3}})^{\sqrt{2}} \div (a^{-\sqrt{54}})$ 를 간단히 하면?

- ① $a^{\sqrt{\frac{7}{2}}}$ ② $a^{\sqrt{2}}$ ③ $a^{-\sqrt{16}}$ ④ $a^{5\sqrt{6}}$ ⑤ a^{36}

해설

지수를 따로 써 보면

$$2\sqrt{3} \times \sqrt{2} + \sqrt{54} = 2\sqrt{6} + 3\sqrt{6}$$

$$= 5\sqrt{6}$$

$$\therefore a^{5\sqrt{6}}$$

4. $9^{\frac{2}{3}} \div 12^{\frac{1}{3}} \times 108^{\frac{1}{3}}$ 을 간단히 하면?

① $\sqrt{2}$

② $\sqrt{3}$

③ 3

④ 6

⑤ 9

해설

$$\begin{aligned} & 9^{\frac{2}{3}} \div 12^{\frac{1}{3}} \times 108^{\frac{1}{3}} \\ &= (3^2)^{\frac{2}{3}} \div (2^2 \times 3)^{\frac{1}{3}} \times (2^2 \times 3^3)^{\frac{1}{3}} \\ &= 3^{\frac{4}{3}} \div (2^{\frac{2}{3}} \times 3^{\frac{1}{3}}) \times (2^{\frac{2}{3}} \times 3^{\frac{3}{3}}) \\ &= 2^{-\frac{2}{3} + \frac{2}{3}} \times 3^{\frac{4}{3} - \frac{1}{3} + \frac{3}{3}} \\ &= 2^0 \times 3^2 \\ &= 9 \end{aligned}$$

5. 다음 식을 간단히 하면?

$20^{\frac{2}{3}} \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}}$

- ① $2\sqrt{2}$
- ② 2
- ③ $\sqrt{5}$
- ④ 5
- ⑤ $\sqrt{20}$

해설

$20^{\frac{2}{3}} = (4 \times 5)^{\frac{2}{3}} = 4^{\frac{2}{3}} \times 5^{\frac{2}{3}}$ 이므로
 $20^{\frac{2}{3}} \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}} = (4^{\frac{2}{3}} \times 5^{\frac{2}{3}}) \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}}$
 $= (4^{\frac{2}{3}} \times 4^{-\frac{2}{3}}) \times (5^{\frac{2}{3}} \times 5^{-\frac{1}{6}})$
 $= 4^{\frac{2}{3} + (-\frac{2}{3})} \times (5^{\frac{2}{3}} \times 5^{-\frac{1}{6}})$
 $= 4^{\frac{2}{3} + (-\frac{2}{3})} \times 5^{\frac{2}{3} + (-\frac{1}{6})}$
 $= 4^0 \times 5^{\frac{1}{2}} = 1 \times \sqrt{5} = \sqrt{5}$

6. 양의 실수 a 에 대하여 $\frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\frac{\sqrt[3]{a}}{\sqrt{a}}} \div \sqrt[3]{\frac{\sqrt[5]{a}}{\sqrt{a}}}$ 의 값은? (단, $a \neq 1$)

- ① $\sqrt[15]{a}$ ② $\frac{1}{\sqrt[15]{a}}$ ③ 1 ④ $\frac{1}{\sqrt[15]{a}}$ ⑤ $\sqrt[15]{a}$

해설

$$\begin{aligned} \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\frac{\sqrt[3]{a}}{\sqrt{a}}} \div \sqrt[3]{\frac{\sqrt[5]{a}}{\sqrt{a}}} &= \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\frac{\sqrt[3]{a}}{\sqrt{a}}} \times \sqrt[3]{\frac{\sqrt{a}}{\sqrt[5]{a}}} \\ &= \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \frac{\sqrt[5]{\sqrt[3]{a}}}{\sqrt[5]{\sqrt{a}}} \times \frac{\sqrt[3]{\sqrt{a}}}{\sqrt[3]{\sqrt[5]{a}}} = \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \frac{\sqrt[15]{a}}{\sqrt[15]{a}} \times \frac{\sqrt[15]{a}}{\sqrt[15]{a}} = \frac{1}{\sqrt[15]{a}} \end{aligned}$$

7. $\sqrt[6]{\frac{\sqrt{2^4}}{\sqrt[3]{5}}} \times \sqrt[9]{\frac{\sqrt{5}}{\sqrt[3]{2^6}}}$ 를 간단히 하여 $\sqrt[n]{4}$ 로 나타낼 때, 자연수 n 의 값은?

- ① 4 ② 6 ③ 9 ④ 12 ⑤ 18

해설

$$\begin{aligned} & \sqrt[6]{\frac{\sqrt{2^4}}{\sqrt[3]{5}}} \times \sqrt[9]{\frac{\sqrt{5}}{\sqrt[3]{2^6}}} = \frac{\sqrt[6]{\sqrt{2^4}}}{\sqrt[6]{\sqrt[3]{5}}} \times \frac{\sqrt[9]{\sqrt{5}}}{\sqrt[9]{\sqrt[3]{2^6}}} \\ &= \frac{\sqrt[12]{2^4}}{\sqrt[18]{5}} \times \frac{\sqrt[18]{5}}{\sqrt[27]{2^6}} \\ &= \frac{\sqrt[3 \times 4]{2^4}}{\sqrt[9 \times 3]{2^{3 \times 2}}} = \frac{\sqrt[3]{2}}{\sqrt[9]{2^2}} \\ &= \frac{\sqrt[9]{2^3}}{\sqrt[9]{2^2}} = \sqrt[9]{\frac{2^3}{2^2}} \\ &= \sqrt[9]{2} = \sqrt[18]{4} \\ &\therefore n = 18 \end{aligned}$$

8. 등식 $\sqrt[4]{a\sqrt{\sqrt[3]{a^2}}}=27$ 을 만족하는 양수 a 의 값은?

① 3

② 3^2

③ 3^3

④ 3^6

⑤ 3^9

해설

$$\begin{aligned}\sqrt[4]{a\sqrt{\sqrt[3]{a^2}}} &= \left\{a\left(a^{\frac{2}{3}}\right)^{\frac{1}{2}}\right\}^{\frac{1}{4}} \\ &= \left(a \cdot a^{\frac{2}{3} \cdot \frac{1}{2}}\right)^{\frac{1}{4}} \\ &= \left(a^{\frac{4}{3}}\right)^{\frac{1}{4}} = a^{\frac{1}{3}}\end{aligned}$$

$$a^{\frac{1}{3}} = 3^3 \text{ 이므로 } \left(a^{\frac{1}{3}}\right)^3 = (3^3)^3$$

$$\therefore a = 3^9$$

9. $a^{\frac{1}{2}} \times a^{-\frac{1}{3}} \div a^{\frac{3}{2}}$ 을 간단히 하면?

① $a \sqrt[3]{a}$

② $a \sqrt{a}$

③ $\frac{1}{a \sqrt[3]{a^2}}$

④ $\frac{1}{a \sqrt{a}}$

⑤ $\frac{1}{a \sqrt[3]{a}}$

해설

$$\begin{aligned} a^{\frac{1}{2}-\frac{1}{3}-\frac{3}{2}} &= a^{\frac{3-2-9}{6}} \\ &= a^{-\frac{8}{6}} = a^{-\frac{4}{3}} = \frac{1}{a \sqrt[3]{a}} \end{aligned}$$

10. $x = 2$ 일 때, $(x^x)^{x^x}$ 는?

① 16

② 64

③ 256

④ 1024

⑤ 65536

해설

$$(2^2)^{2^2} = (2^2)^4 = 2^{16}$$

$$2^{10} = 1024, 2^6 = 64 \text{ 이므로}$$

$$\therefore 2^{16} = 1024 \times 64 = 65536$$

11. 실수 a, b, c, d 에 대하여 $2^a = c, 2^b = d$ 일 때, 4^{a+b} 와 같은 것은?

- ① $\frac{1}{cd}$ ② $\frac{1}{2cd}$ ③ $\frac{1}{c^2d}$ ④ cd ⑤ c^2d^2

해설

$$4^{a+b} = (2^2)^{a+b} = 2^{2a} \cdot 2^{2b} = (2^a)^2 \cdot (2^b)^2 = c^2d^2$$

12. $4^{x-1} = a$ 일 때, $\left(\frac{1}{32}\right)^{1-x}$ 을 a 에 대한 식으로 나타낸 것은?

- ① $\sqrt{a}$ ② $a\sqrt[5]{a}$ ③ $\sqrt[5]{a}$ ④ $\sqrt[5]{a^2}$ ⑤ $a^2\sqrt{a}$

해설

$$4^{x-1} = 2^{2(x-1)} = a \text{ 이므로}$$

$$2^{x-1} = a^{\frac{1}{2}}$$

$$\begin{aligned}\left(\frac{1}{32}\right)^{1-x} &= (2^{-5})^{1-x} = 2^{5(x-1)} \\ &= (2^{x-1})^5 = (a^{\frac{1}{2}})^5 = a^{\frac{5}{2}} = a^2\sqrt{a}\end{aligned}$$

13. 임의의 실수 x 의 네제곱근 중에서 실수인 것의 개수를 $f(x)$ 라 할 때, $f(2^{-2}) + f(-2^2) + f(2^0)$ 의 값은?

① 2

② 3

③ 4

④ 5

⑤ 6

해설

$2^{-2} = \frac{1}{4} > 0$, $-2^2 = -4 < 0$, $2^0 = 1 > 0$ 이고 4는 짝수이므로

$$f(2^{-2}) + f(-2^2) + f(2^0)$$

$$= 2 + 0 + 2$$

$$4$$

14. 서로소인 두 자연수 a, b 에 대하여 $\frac{\sqrt{\sqrt{3}}}{\sqrt{3}} \times \sqrt[3]{3} = 3^{\frac{b}{a}}$ 일 때, $a + b$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 13

해설

$$\frac{\sqrt{\sqrt{3}}}{\sqrt{3}} \times \sqrt[3]{3} = \frac{3^{\frac{1}{4}}}{3^{\frac{1}{2}}} \times 3^{\frac{1}{3}} = 3^{\frac{1}{4} - \frac{1}{2} + \frac{1}{3}} = 3^{\frac{1}{12}}$$

따라서 $a + b = 13$ 이다.

15. $2^{\sqrt{3-2\sqrt{2}}} \times \left(\frac{1}{2}\right)^{\sqrt{3+2\sqrt{2}}}$ 의 값은?

- ① $\frac{1}{4}$ ② $\frac{1}{2}$ ③ 1 ④ 2 ⑤ 4

해설

$$\begin{aligned} 2^{\sqrt{3-2\sqrt{2}}} \times \left(\frac{1}{2}\right)^{\sqrt{3+2\sqrt{2}}} &= 2^{\sqrt{2}-1} \times \left(\frac{1}{2}\right)^{\sqrt{2}+1} \\ &= 2^{\sqrt{2}-1} \times 2^{-\sqrt{2}-1} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} \end{aligned}$$

16. $a = 2^{12}$ 일 때, $\sqrt{\frac{\sqrt[3]{a}}{\sqrt[4]{a}}} \times \sqrt[4]{\frac{\sqrt{a}}{\sqrt[3]{a}}}$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 2

해설

$$(a^{\frac{1}{3}-\frac{1}{4}})^{\frac{1}{2}} \times (a^{\frac{1}{2}-\frac{1}{3}})^{\frac{1}{4}} = a^{\frac{1}{24}} \times a^{\frac{1}{24}} = a^{\frac{1}{12}}$$

$a = 2^{12}$ 이므로

$$a^{\frac{1}{12}} = (2^{12})^{\frac{1}{12}} = 2$$

17. $\frac{\sqrt[3]{8} + \sqrt[3]{3}}{\sqrt[3]{16} + \sqrt[3]{6}}$ 을 간단히 하면?

- ① $\frac{1}{\sqrt[3]{2}}$ ② $\frac{1}{\sqrt[3]{3}}$ ③ $\frac{1}{2\sqrt[3]{2}}$ ④ $\frac{1}{3\sqrt[3]{3}}$ ⑤ $\frac{1}{\sqrt[3]{6}}$

해설

$$\begin{aligned}\frac{\sqrt[3]{8} + \sqrt[3]{3}}{\sqrt[3]{16} + \sqrt[3]{6}} &= \frac{\sqrt[3]{2^3} + \sqrt[3]{3}}{\sqrt[3]{2^3 \cdot 2} + \sqrt[3]{3 \cdot 2}} \\&= \frac{2 + \sqrt[3]{3}}{2 + \sqrt[3]{3}} \\&= \frac{\sqrt[3]{2}(2 + \sqrt[3]{3})}{\sqrt[3]{2}(2 + \sqrt[3]{3})} \\&= \frac{1}{\sqrt[3]{2}}\end{aligned}$$

18. 세 수 $A = \sqrt[3]{\sqrt{100}}$, $B = \sqrt{5}$, $C = \sqrt[3]{\sqrt{121}}$ 의 대소 관계를 바르게 나타낸 것은?

- ① $A < B < C$ ② $A < C < B$ ③ $B < A < C$
④ $B < C < A$ ⑤ $C < A < B$

해설

$$A = \sqrt[3]{\sqrt{100}} = \sqrt[3]{100^{\frac{1}{2}}} = 100^{\frac{1}{6}}$$

$$B = \sqrt{5} = 5^{\frac{1}{2}} = (5^3)^{\frac{1}{6}} = 125^{\frac{1}{6}}$$

$$C = \sqrt[3]{\sqrt{121}} = \sqrt[3]{121^{\frac{1}{2}}} = 121^{\frac{1}{6}}$$

이므로 A, B, C 의 대소 관계는 $A < C < B$ 이다

19. $(7^{\frac{1}{4}} - 5^{\frac{1}{4}})(7^{\frac{1}{4}} + 5^{\frac{1}{4}})(7^{\frac{1}{2}} + 5^{\frac{1}{2}})$ 의 값은?

- ① 2 ② 6 ③ 10 ④ 14 ⑤ 18

해설

$$\begin{aligned} & (7^{\frac{1}{4}} - 5^{\frac{1}{4}})(7^{\frac{1}{4}} + 5^{\frac{1}{4}})(7^{\frac{1}{2}} + 5^{\frac{1}{2}}) \\ & \{ (7^{\frac{1}{4}})^2 - (5^{\frac{1}{4}})^2 \} (7^{\frac{1}{2}} + 5^{\frac{1}{2}}) \\ & = (7^{\frac{1}{2}} - 5^{\frac{1}{2}})(7^{\frac{1}{2}} + 5^{\frac{1}{2}}) = (7^{\frac{1}{2}})^2 - (5^{\frac{1}{2}})^2 \\ & = 7 - 5 = 2 \end{aligned}$$

20. $x > 0$ 이고 $x + x^{-1} = 3$ 일 때, $x^{\frac{3}{2}} + x^{-\frac{3}{2}}$ 의 값은?

- ① $\sqrt{5}$ ② $2\sqrt{5}$ ③ $3\sqrt{5}$ ④ $4\sqrt{5}$ ⑤ $5\sqrt{5}$

해설

$$\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = x + x^{-1} + 2 \text{에서}$$

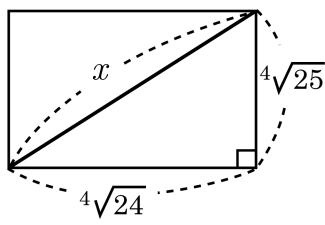
$$\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = 3 + 2 = 5 \text{이므로}$$

$$\text{이 때, } x^{\frac{1}{2}} + x^{-\frac{1}{2}} > 0 \text{이므로 } x^{\frac{1}{2}} + x^{-\frac{1}{2}} = \sqrt{5}$$

$$\therefore x^{\frac{3}{2}} + x^{-\frac{3}{2}} = \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^3 - 3 \cdot x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$$

$$= (\sqrt{5})^3 - 3\sqrt{5} = 5\sqrt{5} - 3\sqrt{5} = 2\sqrt{5}$$

25. 가로와 세로의 길이가 각각 $\sqrt[4]{24}$, $\sqrt[4]{25}$ 인 직사각형의 대각선의 길이는?



- ① $\sqrt{5} + \sqrt{2}$ ② $\sqrt{5} - \sqrt{2}$ ③ 3
 ④ $\sqrt{3} - \sqrt{2}$ ⑤ $\sqrt{3} + \sqrt{2}$

해설

$$\begin{aligned} x &= \sqrt{(\sqrt[4]{24})^2 + (\sqrt[4]{25})^2} \\ &= \sqrt{\sqrt{24} + \sqrt{25}} \\ &= \sqrt{2\sqrt{6} + 5} \\ &= \sqrt{(\sqrt{3} + \sqrt{2})^2} \\ \therefore x &= \sqrt{3} + \sqrt{2} \end{aligned}$$