1. -8의 세제곱근 중에서 실수를 a, 16의 네제곱근 중에서 실수를 b라 할 때, $a+b^2$의 값을 구하면?

① 0 ② 2 ③ 4 ④ 6 ⑤ 8

해설

$a=-2$

$b=2\ or\ -2$

$a+b^2=-2+4=2$

2. $\sqrt[3]{9^4} \div \sqrt{3^3} \times \sqrt[6]{\frac{1}{3}}$ 의 값을 구하면?

① 9 ② 3 ③ $\sqrt{3}$ ④ $\frac{\sqrt{3}}{3}$ ⑤ $\frac{1}{3}$

해설

$$\begin{aligned}\sqrt[3]{9^4} \div \sqrt{3^3} \times \sqrt[6]{\frac{1}{3}} &= (3^2)^{\frac{4}{3}} \div 3^{\frac{3}{2}} \times (3^{-1})^{\frac{1}{6}} \\ &= 3^{\frac{8}{3}-\frac{3}{2}-\frac{1}{6}} \\ &= 3\end{aligned}$$

3. 양수 a에 대하여 $(a^{2\sqrt{3}})^{\sqrt{2}} \div (a^{-\sqrt{54}})$를 간단히 하면?

① $a^{\sqrt{\frac{3}{2}}}$ ② $a^{\sqrt{2}}$ ③ $a^{-\sqrt{16}}$ ④ $a^{5\sqrt{6}}$ ⑤ a^{36}

해설

지수를 따로 써 보면

$2\sqrt{3} \times \sqrt{2} + \sqrt{54} = 2\sqrt{6} + 3\sqrt{6}$

$= 5\sqrt{6}$

$\therefore\ a^{5\sqrt{6}}$

4. $9^{\frac{2}{3}} \div 12^{\frac{1}{3}} \times 108^{\frac{1}{3}}$ 을 간단히 하면?

① $\sqrt{2}$ ② $\sqrt{3}$ ③ 3 ④ 6 ⑤ 9

해설

$$9^{\frac{2}{3}} \div 12^{\frac{1}{3}} \times 108^{\frac{1}{3}}$$
$$= (3^2)^{\frac{2}{3}} \div (2^2 \times 3)^{\frac{1}{3}} \times (2^2 \times 3^3)^{\frac{1}{3}}$$
$$= 3^{\frac{4}{3}} \div \left(2^{\frac{2}{3}} \times 3^{\frac{1}{3}}\right) \times \left(2^{\frac{2}{3}} \times 3^{\frac{3}{3}}\right)$$
$$= 2^{-\frac{2}{3}+\frac{2}{3}} \times 3^{\frac{4}{3}-\frac{1}{3}+\frac{3}{3}}$$
$$= 2^0 \times 3^2$$
$$= 9$$

5. 다음 식을 간단히 하면?

$$20^{\frac{2}{3}} \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}}$$

① $2\sqrt{2}$ ② 2 ③ $\sqrt{5}$ ④ 5 ⑤ $\sqrt{20}$

해설

$20^{\frac{2}{3}} = (4 \times 5)^{\frac{2}{3}} = 4^{\frac{2}{3}} \times 5^{\frac{2}{3}}$ 이므로

$20^{\frac{2}{3}} \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}} = (4^{\frac{2}{3}} \times 5^{\frac{2}{3}}) \times 4^{-\frac{2}{3}} \times 5^{-\frac{1}{6}}$

$= (4^{\frac{2}{3}} \times 4^{-\frac{2}{3}}) \times (5^{\frac{2}{3}} \times 5^{-\frac{1}{6}})$

$= 4^{\frac{2}{3}+(-\frac{2}{3})} \times (5^{\frac{2}{3}} \times 5^{-\frac{1}{6}})$

$= 4^{\frac{2}{3}+(-\frac{2}{3})} \times 5^{\frac{2}{3}+(-\frac{1}{6})}$

$= 4^0 \times 5^{\frac{1}{2}} = 1 \times \sqrt{5} = \sqrt{5}$

6. 양의 실수 a에 대하여 $\dfrac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\dfrac{\sqrt[3]{a}}{\sqrt{a}}} \div \sqrt[3]{\dfrac{\sqrt[5]{a}}{\sqrt{a}}}$의 값은?(단, $a \neq 1$)

① $\sqrt[10]{a}$ ② $\dfrac{1}{\sqrt[10]{a}}$ ③ 1 ④ $\dfrac{1}{\sqrt[15]{a}}$ ⑤ $\sqrt[10]{a}$

해설

$$\frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\frac{\sqrt[3]{a}}{\sqrt{a}}} \div \sqrt[3]{\frac{\sqrt[5]{a}}{\sqrt{a}}} = \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \sqrt[5]{\frac{\sqrt[3]{a}}{\sqrt{a}}} \times \sqrt[3]{\frac{\sqrt{a}}{\sqrt[5]{a}}}$$

$$= \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \frac{\sqrt[5]{\sqrt[3]{a}}}{\sqrt[5]{\sqrt{a}}} \times \frac{\sqrt[3]{\sqrt{a}}}{\sqrt[3]{\sqrt[5]{a}}} = \frac{\sqrt[5]{a}}{\sqrt[3]{a}} \times \frac{\sqrt[15]{a}}{\sqrt[10]{a}} \times \frac{\sqrt[6]{a}}{\sqrt[15]{a}} = \frac{1}{\sqrt[15]{a}}$$

7. $\sqrt[6]{\dfrac{\sqrt{2^4}}{\sqrt[3]{5}}} \times \sqrt[9]{\dfrac{\sqrt{5}}{\sqrt[3]{2^6}}}$ 를 간단히 하여 $\sqrt[n]{4}$ 로 나타낼 때, 자연수 n 의 값은?

① 4 ② 6 ③ 9 ④ 12 ⑤ 18

해설

$$\sqrt[6]{\frac{\sqrt{2^4}}{\sqrt[3]{5}}} \times \sqrt[9]{\frac{\sqrt{5}}{\sqrt[3]{2^6}}} = \frac{\sqrt[6]{\sqrt{2^4}}}{\sqrt[6]{\sqrt[3]{5}}} \times \frac{\sqrt[9]{\sqrt{5}}}{\sqrt[9]{\sqrt[3]{2^6}}}$$

$$= \frac{\sqrt[12]{2^4}}{\sqrt[18]{5}} \times \frac{\sqrt[18]{5}}{\sqrt[27]{2^6}}$$

$$= \frac{\sqrt[3\times4]{2^4}}{\sqrt[9\times3]{2^{3\times2}}} = \frac{\sqrt[3]{2}}{\sqrt[9]{2^2}}$$

$$= \frac{\sqrt[9]{2^3}}{\sqrt[9]{2^2}} = \sqrt[9]{\frac{2^3}{2^2}}$$

$$= \sqrt[9]{2} = \sqrt[18]{4}$$

$\therefore n = 18$

8. 등식 $\sqrt[4]{a\sqrt{\sqrt[3]{a^2}}} = 27$을 만족하는 양수 a의 값은?

① 3 ② 3^2 ③ 3^3 ④ 3^6 ⑤ 3^9

해설

$$\sqrt[4]{a\sqrt{\sqrt[3]{a^2}}} = \left\{a\left(a^{\frac{2}{3}}\right)^{\frac{1}{2}}\right\}^{\frac{1}{4}}$$
$$= \left(a \cdot a^{\frac{2}{3}\cdot\frac{1}{2}}\right)^{\frac{1}{4}}$$
$$= \left(a^{\frac{4}{3}}\right)^{\frac{1}{4}} = a^{\frac{1}{3}}$$

$a^{\frac{1}{3}} = 3^3$ 이므로 $(a^{\frac{1}{3}})^3 = (3^3)^3$

$\therefore a = 3^9$

9. $a^{\frac{1}{2}} \times a^{-\frac{1}{3}} \div a^{\frac{3}{2}}$ 을 간단히 하면?

① $a\sqrt[3]{a}$ ② $a\sqrt{a}$ ③ $\dfrac{1}{a\sqrt[3]{a^2}}$

④ $\dfrac{1}{a\sqrt{a}}$ ⑤ $\dfrac{1}{a\sqrt[3]{a}}$

해설

$a^{\frac{1}{2}-\frac{1}{3}-\frac{3}{2}} = a^{\frac{3-2-9}{6}}$

$= a^{\frac{-8}{6}} = a^{\frac{-4}{3}} = \dfrac{1}{a\sqrt[3]{a}}$

10. $x = 2$ 일 때, $(x^x)^{x^x}$ 는?

① 16　② 64　③ 256

④ 1024　⑤ 65536

해설

$(2^2)^{2^2} = (2^2)^4 = 2^{16}$

$2^{10} = 1024,\ 2^6 = 64$ 이므로

$\therefore\ 2^{16} = 1024 \times 64 = 65536$

11. 실수 $a,\ b,\ c,\ d$에 대하여 $2^a = c,\ 2^b = d$일 때, 4^{a+b}와 같은 것은?

① $\dfrac{1}{cd}$ ② $\dfrac{1}{2cd}$ ③ $\dfrac{1}{c^2d}$ ④ cd ⑤ c^2d^2

해설

$4^{a+b} = (2^2)^{a+b} = 2^{2a} \cdot 2^{2b} = (2^a)^2 \cdot (2^b)^2 = c^2d^2$

12. $4^{x-1}=a$ 일 때, $\left(\frac{1}{32}\right)^{1-x}$ 을 a 에 대한 식으로 나타낸 것은?

① $\sqrt{a}$ ② $a\sqrt[5]{a}$ ③ $\sqrt[5]{a}$ ④ $\sqrt[5]{a^2}$ ⑤ $a^2\sqrt{a}$

해설

$4^{x-1}=2^{2(x-1)}=a$ 이므로

$2^{x-1}=a^{\frac{1}{2}}$

$\left(\frac{1}{32}\right)^{1-x}=(2^{-5})^{1-x}=2^{5(x-1)}$

$=(2^{x-1})^5=(a^{\frac{1}{2}})^5=a^{\frac{5}{2}}=a^2\sqrt{a}$

13. 임의의 실수 x의 네제곱근 중에서 실수인 것의 개수를 $f(x)$라 할 때, $f(2^{-2}) + f(-2^2) + f(2^0)$의 값은?

① 2 ② 3 ③ 4 ④ 5 ⑤ 6

해설

$2^{-2} = \frac{1}{4} > 0,\ -2^2 = -4 < 0,\ 2^0 = 1 > 0$이고 4는 짝수이므로

$f(2^{-2}) + f(-2^2) + f(2^0)$

$= 2 + 0 + 2$

4

14. 서로소인 두 자연수 a, b에 대하여 $\dfrac{\sqrt{\sqrt{3}}}{\sqrt{3}} \times \sqrt[3]{3} = 3^{\frac{b}{a}}$ 일 때, $a+b$의 값을 구하여라.

▶ 답:

▷ 정답: 13

해설

$$\frac{\sqrt{\sqrt{3}}}{\sqrt{3}} \times \sqrt[3]{3} = \frac{3^{\frac{1}{4}}}{3^{\frac{1}{2}}} \times 3^{\frac{1}{3}} = 3^{\frac{1}{4}-\frac{1}{2}+\frac{1}{3}} = 3^{\frac{1}{12}}$$

따라서 $a+b=13$이다.

15. $2^{\sqrt{3-2\sqrt{2}}} \times \left(\frac{1}{2}\right)^{\sqrt{3+2\sqrt{2}}}$ 의 값은?

① $\frac{1}{4}$ ② $\frac{1}{2}$ ③ 1 ④ 2 ⑤ 4

해설

$$2^{\sqrt{3-2\sqrt{2}}} \times \left(\frac{1}{2}\right)^{\sqrt{3+2\sqrt{2}}} = 2^{\sqrt{2}-1} \times \left(\frac{1}{2}\right)^{\sqrt{2}+1}$$

$$= 2^{\sqrt{2}-1} \times 2^{-\sqrt{2}-1} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

16. $a = 2^{12}$ 일 때, $\sqrt{\dfrac{\sqrt[3]{a}}{\sqrt[4]{a}}} \times \sqrt[4]{\dfrac{\sqrt{a}}{\sqrt[3]{a}}}$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 2

해설

$(a^{\frac{1}{3}-\frac{1}{4}})^{\frac{1}{2}} \times (a^{\frac{1}{2}-\frac{1}{3}})^{\frac{1}{4}} = a^{\frac{1}{24}} \times a^{\frac{1}{24}} = a^{\frac{1}{12}}$

$a = 2^{12}$ 이므로

$a^{\frac{1}{12}} = (2^{12})^{\frac{1}{12}} = 2$

17. $\dfrac{\sqrt[3]{8}+\sqrt[3]{3}}{\sqrt[3]{16}+\sqrt[3]{6}}$ 을 간단히 하면?

① $\dfrac{1}{\sqrt[3]{2}}$ ② $\dfrac{1}{\sqrt[3]{3}}$ ③ $\dfrac{1}{2\sqrt[3]{2}}$ ④ $\dfrac{1}{3\sqrt[3]{3}}$ ⑤ $\dfrac{1}{\sqrt[3]{6}}$

해설

$$\begin{aligned}\frac{\sqrt[3]{8}+\sqrt[3]{3}}{\sqrt[3]{16}+\sqrt[3]{6}} &= \frac{\sqrt[3]{2^3}+\sqrt[3]{3}}{\sqrt[3]{2^3\cdot 2}+\sqrt[3]{3\cdot 2}} \\ &= \frac{2+\sqrt[3]{3}}{2\sqrt[3]{2}+\sqrt[3]{3}\sqrt[3]{2}} \\ &= \frac{2+\sqrt[3]{3}}{\sqrt[3]{2}(2+\sqrt[3]{3})} \\ &= \frac{1}{\sqrt[3]{2}}\end{aligned}$$

18. 세 수 $A = \sqrt[3]{\sqrt{100}}, B = \sqrt{5}, C = \sqrt[3]{\sqrt{121}}$ 의 대소 관계를 바르게 나타낸 것은?

① $A < B < C$ ② $A < C < B$ ③ $B < A < C$

④ $B < C < A$ ⑤ $C < A < B$

해설

$A = \sqrt[3]{\sqrt{100}} = \sqrt[3]{100^{\frac{1}{2}}} = 100^{\frac{1}{6}}$

$B = \sqrt{5} = 5^{\frac{1}{2}} = (5^3)^{\frac{1}{6}} = 125^{\frac{1}{6}}$

$C = \sqrt[3]{\sqrt{121}} = \sqrt[3]{121^{\frac{1}{2}}} = 121^{\frac{1}{6}}$

이므로 A, B, C 의 대소 관계는 $A < C < B$ 이다

19. $(7^{\frac{1}{4}}-5^{\frac{1}{4}})(7^{\frac{1}{4}}+5^{\frac{1}{4}})(7^{\frac{1}{2}}+5^{\frac{1}{2}})$의 값은?

① 2 ② 6 ③ 10 ④ 14 ⑤ 18

해설

$(7^{\frac{1}{4}}-5^{\frac{1}{4}})(7^{\frac{1}{4}}+5^{\frac{1}{4}})(7^{\frac{1}{2}}+5^{\frac{1}{2}})$

$\left\{(7^{\frac{1}{4}})^2-(5^{\frac{1}{4}})^2\right\}(7^{\frac{1}{2}}+5^{\frac{1}{2}})$

$=(7^{\frac{1}{2}}-5^{\frac{1}{2}})(7^{\frac{1}{2}}+5^{\frac{1}{2}})=(7^{\frac{1}{2}})^2-(5^{\frac{1}{2}})^2$

$=7-5=2$

20. $x>0$이고 $x+x^{-1}=3$일 때, $x^{\frac{3}{2}}+x^{-\frac{3}{2}}$의 값은?

① $\sqrt{5}$ ② $2\sqrt{5}$ ③ $3\sqrt{5}$ ④ $4\sqrt{5}$ ⑤ $5\sqrt{5}$

해설

$\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2=x+x^{-1}+2$에서

$\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2=3+2=5$이므로

이 때, $x^{\frac{1}{2}}+x^{-\frac{1}{2}}>0$이므로 $x^{\frac{1}{2}}+x^{-\frac{1}{2}}=\sqrt{5}$

$\therefore x^{\frac{3}{2}}+x^{-\frac{3}{2}}=(x^{\frac{1}{2}}+x^{-\frac{1}{2}})^3-3\cdot x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}}(x^{\frac{1}{2}}+x^{-\frac{1}{2}})$

$=(\sqrt{5})^3-3\sqrt{5}=5\sqrt{5}-3\sqrt{5}=2\sqrt{5}$

21. $x > 0$이고 $x^2 + x^{-2} = 7$일 때, $(x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x + x^{-1})$의 값은?

① $\sqrt{7}$ ② $2\sqrt{5}$ ③ $3\sqrt{5}$ ④ $3\sqrt{7}$ ⑤ $7\sqrt{3}$

해설

곱셈 공식을 써서 식을 변형한다.

$x^2 + x^{-2} = 7$

$(x + x^{-1})^2 = x^2 + x^{-2} + 2$에서

$(x + x^{-1})^2 = 7 + 2 = 9$

$x + x^{-1} > 0$이므로 $x + x^{-1} = 3$

$(x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 = x + x^{-1} + 2$에서

$(x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 = 3 + 2 = 5$

$x^{\frac{1}{2}} + x^{-\frac{1}{2}} > 0$이므로 $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = \sqrt{5}$

$\therefore (x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x + x^{-1}) = 3\sqrt{5}$

22. $2^x - 2^{-x} = 2\sqrt{3}$ 일 때, $4^x - 4^{-x}$ 의 값은?

① 4 ② 6 ③ 8 ④ $8\sqrt{3}$ ⑤ $12\sqrt{3}$

해설

$$\begin{aligned} 4^x - 4^{-x} &= (2^x)^2 - (2^{-x})^2 \\ &= (2^x + 2^{-x})(2^x - 2^{-x}) \\ &= 2\sqrt{3}(2^x + 2^{-x}) \end{aligned}$$

한편, $(2^x + 2^{-x})^2 = (2^x - 2^{-x})^2 + 4$

$= (2\sqrt{3})^2 + 4 = 16$

$2^x + 2^{-x} > 0$ 이므로 $2^x + 2^{-x} = 4$

$\therefore 4^x - 4^{-4x} = 2\sqrt{3} \times 4 = 8\sqrt{3}$

23. $2^6 = a,\ 9^4 = b$일 때, 12^5를 $a,\ b$에 관한 식으로 나타내면?

① $a^{\frac{5}{6}}b^{\frac{5}{8}}$ ② $a^{\frac{5}{4}}b^{\frac{5}{4}}$ ③ $a^{\frac{5}{3}}b^{\frac{5}{8}}$ ④ $a^{\frac{5}{3}}b^{\frac{7}{8}}$ ⑤ $a^{\frac{7}{4}}b^{\frac{3}{2}}$

해설

$2^6 = a$에서 $2 = a^{\frac{1}{6}}$

$9^4 = b$에서 $(3^2)^4 = 3^8 = b$

$\therefore 3 = b^{\frac{1}{8}}$

$\therefore\ 12^5 = (2^2 \times 3)^5 = 2^{10} \times 3^5 = (a^{\frac{1}{6}})^{10} \times (b^{\frac{1}{8}})^5 = a^{\frac{5}{3}}b^{\frac{5}{8}}$

24. $a^{2x}=5$ 일 때, $\dfrac{a^{3x}+a^{-3x}}{a^{x}-a^{-x}}$ 의 값은?

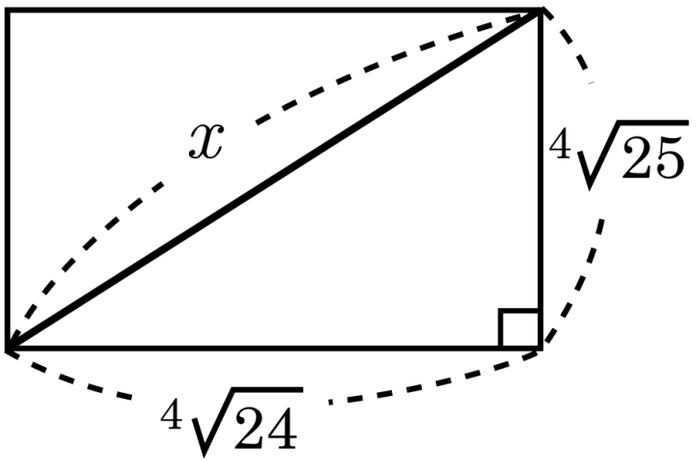
① $\dfrac{21}{5}$ ② $\dfrac{9}{2}$ ③ $\dfrac{31}{5}$ ④ $\dfrac{51}{5}$ ⑤ $\dfrac{63}{10}$

해설

$$\frac{a^{3x}+a^{-3x}}{a^{x}-a^{-x}}=\frac{a^{4x}+a^{-2x}}{a^{2x}-1}$$

$$=\frac{25+\frac{1}{5}}{5-1}=\frac{\frac{126}{5}}{4}=\frac{126}{20}=\frac{63}{10}$$

25. 가로와 세로의 길이가 각각 $\sqrt[4]{24}$, $\sqrt[4]{25}$인 직사각형의 대각선의 길이는?



① $\sqrt{5}+\sqrt{2}$ ② $\sqrt{5}-\sqrt{2}$ ③ 3

④ $\sqrt{3}-\sqrt{2}$ ⑤ $\sqrt{3}+\sqrt{2}$

해설

$$x=\sqrt{(\sqrt[4]{24})^2+(\sqrt[4]{25})^2}$$
$$=\sqrt{\sqrt{24}+\sqrt{25}}$$
$$=\sqrt{2\sqrt{6}+5}$$
$$=\sqrt{(\sqrt{3}+\sqrt{2})^2}$$
$$\therefore x=\sqrt{3}+\sqrt{2}$$