1. $\sum_{i=1}^{100} x_i = 4, \sum_{i=1}^{100} y_i = 6$일때, $\sum_{k=1}^{100}(3x_k - 2y_k)$의 값을 구하여라.

▶ 답:

▷ 정답: 0

해설

$\sum_{k=1}^{100}(3x_k - 2y_k) = 3\sum_{k=1}^{100} x_k - 2\sum_{k=1}^{100} y_k$
$= 3\sum_{i=1}^{100} x_i - 2\sum_{i=1}^{100} y_i = 3 \cdot 4 - 2 \cdot 6 = 0$

2. $\sum_{k=1}^{5} a_k = 5$, $\sum_{k=1}^{5} b_k = 7$ 일 때, $\sum_{k=1}^{5}(3a_k + 2b_k)$ 의 값은?

① 21 ② 22 ③ 23 ④ 24 ⑤ 29

해설

$\sum_{k=1}^{5}(3a_k + 2b_k) = \sum_{k=1}^{5} 3a_k + \sum_{k=1}^{5} 2b_k$
$= 3\sum_{k=1}^{5} a_k + 2\sum_{k=1}^{5} b_k$
$= 3 \times 5 + 2 \times 7 = 15 + 14 = 29$

3. 다음 중 옳은 것은?

① $1+4+7+\cdots+(3n-5)=\sum_{k=1}^{n}(3k-5)$

② $2+4+6+\cdots+2(n+1)=\sum_{k=1}^{n}2(k+1)$

③ $3+5+7+\cdots+(2n-1)=\sum_{k=1}^{n}(2k+1)$

④ $4+5+6+\cdots+(n+3)=\sum_{k=1}^{n}(k+3)$

⑤ $3+4+5+\cdots+n=\sum_{k=1}^{n}k$

해설

① $1+4+7+\cdots+(3n-5)=\sum_{k=1}^{n-1}(3k-2)$
② $2+4+6+\cdots+2(n+1)=\sum_{k=1}^{n+1}2n$
③ $3+5+7+\cdots+(2n-1)=\sum_{k=1}^{n-1}(2k+1)$
⑤ $3+4+5+\cdots+n=\sum_{k=1}^{n-2}(k+2)$

4. $\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 470

해설

$\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2 = \left(\sum_{k=1}^{15} k^2 - \sum_{k=1}^{10} k^2\right) - \sum_{k=1}^{10} k^2$

$= \sum_{k=1}^{15} k^2 - 2\sum_{k=1}^{10} k^2$

$= \frac{15 \cdot 16 \cdot 31}{6} - 2 \cdot \frac{10 \cdot 11 \cdot 21}{6} = 470$

5. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$ 의 값은?

① 385　② 550　③ 1100　④ 1150　⑤ 1200

해설

$\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$

$=\sum_{j=1}^{10}\left\{3j+\frac{j(j+1)}{2}\right\}$

$=\sum_{j=1}^{10}\left(\frac{j^2+7i}{2}\right)$

$=\frac{1}{2}\left(\sum_{j=1}^{10}j^2+7\sum_{j=1}^{10}j\right)$

$=\frac{1}{2}\left(\frac{10\cdot 11\cdot 21}{6}+7\times\frac{10\cdot 11}{2}\right)$

$=\frac{1}{2}(385+385)=385$

6. $\sum_{k=1}^{n} a_k = n^2 + 2n$ 일 때, $\sum_{k=1}^{3}(a_k + 1)^2 - \sum_{k=1}^{3}(a_k - 1)^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 60

해설

$\sum_{k=1}^{3}(a_k + 1)^2 - \sum_{k=1}^{3}(a_k - 1)^2$
$= \sum_{k=1}^{3}(a_k + 2a_k + 1) - \sum_{k=1}^{3}(a_k^2 - 2a_k + 1)$
$= 4\sum_{k=1}^{3} a_k = 4(3^2 + 2 \times 3) = 60$

7. $\sum_{l=1}^{n}\left(\sum_{k=1}^{l}k\right)=364$를 만족하는 n의 값은?

① 10 ② 11 ③ 12 ④ 13 ⑤ 14

해설

$\sum_{l=1}^{n}\left(\sum_{k=1}^{l}k\right)=\sum_{l=1}^{n}\left\{\frac{l(l+1)}{2}\right\}=\frac{1}{2}\sum_{l=1}^{n}(l^2+l)$

$=\frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right\}$

$=\frac{1}{2}\times\frac{n(n+1)(n+2)}{3}$

$=\frac{n(n+1)(n+2)}{6}$

$=364=2^2\times7\times13$

$\therefore\ n(n+1)(n+2)=6\times2^2\times7\times13=12\times13\times14$

따라서 $n=12$

8. 수열 $2\cdot3,\ 3\cdot5,\ 4\cdot7,\ 5\cdot9,\ \cdots$ 의 제 n항까지의 합은?

① $4n^2+15n+17$
② $n(4n^2+15n+17)$
③ $\dfrac{4n^2+15n+17}{3}$
④ $\dfrac{n(4n^2+15n+17)}{3}$
⑤ $\dfrac{n(4n^2+15n+17)}{6}$

해설

$a_k=(k+1)(2k+1)=2k^2+3k+1$ 이므로

$S_n=\sum_{k=1}^{n}(2k^2+3k+1)$

$=2\sum_{k=1}^{n}k^2+3\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$

$=2\cdot\dfrac{n(n+1)(2n+1)}{6}+3\cdot\dfrac{n(n+1)}{2}+n$

$=\dfrac{n(4n^2+15n+17)}{6}$

9. 다음 수열의 첫째항부터 제 10항까지의 합은?

$1\cdot1\cdot3,\ 2\cdot3\cdot5,\ 3\cdot5\cdot7,\ 4\cdot7\cdot9,\ \cdots$

① 10050 ② 11000 ③ 11055
④ 12045 ⑤ 12100

해설

주어진 수열의 일반항은 $n(2n-1)(2n+1)=4n^3-n$이므로
첫째항부터 제 10항까지의 합은
$\sum_{k=1}^{10}(4k^3-k)=4\cdot\left(\frac{10\times11}{2}\right)^2-\frac{10\times11}{2}$
$=12100-55=12045$

10. $1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1$ 의 값은?

① 1310 ② 1320 ③ 1330 ④ 1340 ⑤ 1350

해설

$1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1$

$= 1 \cdot (20-1) + 2 \cdot (20-2) + 3 \cdot (20-3) + \cdots + 19 \cdot (20-19)$

$= \sum_{k=1}^{19} k(20-k) = \sum_{k=1}^{19}(20k - k^2)$

$= 20 \times \dfrac{19 \cdot 20}{2} - \dfrac{19 \cdot 20 \cdot 39}{6}$

$= 190(20-13) = 1330$

11. 2^n 을 3으로 나눈 나머지를 a_n 이라 할 때, $\sum_{k=1}^{12} a_k$ 의 값은?

① 16　② 17　③ 18　④ 19　⑤ 20

해설

$a_1 = 2^1$ 을 3으로 나눈 나머지 $a_1 = 2$
$a_2 = 2^2$ 을 3으로 나눈 나머지 $a_2 = 1$
$a_3 = 2^3$ 을 3으로 나눈 나머지 $a_3 = 2$
$a_4 = 2^4$ 을 3으로 나눈 나머지 $a_4 = 1$
$a_5 = 2^5$ 을 3으로 나눈 나머지 $a_5 = 2$
즉 a_n 은 n 이 홀수일 때는 2
n 이 짝수일 때는 1
$\sum_{k=1}^{12} a_k = 6 \cdot (2+1) = 6 \cdot 3 = 18$

12. 수열 $\{a_n\}$이 $\sum_{k=1}^{n} a_{2k-1} = n^2$, $\sum_{k=1}^{n} a_{2k} = 2^n$을 만족할 때, $a_9 + a_{10}$의 값은?

① 20 ② 22 ③ 25 ④ 27 ⑤ 30

해설

$n \geq 2$일 때,

$a_{2n-1} = \sum_{k=1}^{n} a_{2k-1} - \sum_{k=1}^{n-1} a_{2k-1} = n^2 - (n-1)^2 = 2n - 1$

$\therefore\ a_9 = 2 \cdot 5 - 1 = 9$

$a_{2n} = \sum_{k=1}^{n} a_{2k} - \sum_{k=1}^{n-1} a_{2k} = 2^n - 2^{n-1} = 2^{n-1}$

$\therefore\ a_{10} = 2^{5-1} = 16$

$\therefore\ a_9 + a_{10} = 25$

13. $\sum_{k=1}^{n} \frac{1}{\sqrt{k}+\sqrt{k+1}}$ 의 값은?

① $\sqrt{n-1}-1$　　② $\sqrt{n+1}-1$　　③ $\sqrt{n+1}$

④ $\sqrt{n+1}+1$　　⑤ $\sqrt{2n+1}+1$

해설

$$\frac{1}{\sqrt{k}+\sqrt{k+1}} = \frac{\sqrt{k+1}-\sqrt{k}}{(\sqrt{k+1}+\sqrt{k})(\sqrt{k+1}-\sqrt{k})}$$

$$= \frac{\sqrt{k+1}-\sqrt{k}}{(k+1)-k} = \sqrt{k+1}-\sqrt{k}$$

따라서

(주어진 식)$=\sum_{k=1}^{n}(\sqrt{k+1}-\sqrt{k})$

$= (\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{n+1}-\sqrt{n})$

$= \sqrt{n+1}-1$

14. $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)}$ 의 값은?

① $\frac{1}{n+1}$ ② $\frac{2n}{n+1}$ ③ $\frac{n}{2n+1}$
④ $\frac{n}{n+2}$ ⑤ $\frac{2n}{2n+1}$

해설

(준 식)$= \frac{1}{2} \sum_{k=1}^{n} \left\{ \frac{1}{2k-1} - \frac{1}{2k+1} \right\}$

$= \frac{1}{2} \cdot \left\{ \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \cdots \right\}$

$+ \frac{1}{2} \left\{ \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \right\}$

$= \frac{1}{2} \left(1 - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$

15. 함수 $f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$에 대하여 $\sum_{k=1}^{20} \frac{2k+1}{f(k)}$의 값은?

① $\frac{40}{7}$ ② $\frac{45}{8}$ ③ $\frac{17}{3}$ ④ $\frac{57}{10}$ ⑤ $\frac{63}{11}$

해설

$f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$= \sum_{k=1}^{20} k^2 = \frac{n(n+1)(2n+1)}{6}$이므로

$\sum_{k=1}^{20} \frac{2k+1}{f(x)} = \sum_{k=1}^{20} \frac{2k+1}{\frac{k(k+1)(2k+1)}{6}}$

$= \sum_{k=1}^{20} \frac{6}{k(k+1)} = 6\sum_{k=1}^{20} \left(\frac{1}{k} - \frac{1}{k+1}\right)$

$= 6\left(1 - \frac{1}{21}\right) = 6 \times \frac{20}{21} = \frac{40}{7}$

16. x에 대한 이차방정식 $x^2+4x-(2n-1)(2n+1)=0$의 두 근 α_n, β_n에 대하여 $\sum_{k=1}^{10}\left(\frac{1}{\alpha_n}+\frac{1}{\beta_n}\right)$의 값은?

① $\frac{11}{21}$ ② $\frac{20}{21}$ ③ $\frac{31}{21}$ ④ $\frac{40}{21}$ ⑤ $\frac{50}{21}$

해설

(준 식)$=\sum_{n=1}^{10}\frac{\alpha_n+\beta_n}{\alpha_n\cdot\beta_n}$

$=\sum_{n=1}^{10}\frac{-4}{-(2n-1)(2n+1)}$

$=4\sum_{n=1}^{10}\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$=2\sum_{n=1}^{10}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$=2\left(1-\frac{1}{21}\right)=\frac{40}{21}$

17. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+n}$ 의 값을 구하면?

① $\frac{n}{n+1}$ ② $\frac{2n}{n+1}$ ③ $\frac{3n}{n+1}$ ④ $\frac{4n}{n+1}$ ⑤ $\frac{5n}{n+1}$

해설

(주어진 식)$=\sum_{k=1}^{n}\frac{1}{\frac{k(k+1)}{2}}$

$=2\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=2\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{n}-\frac{1}{n+1}\right)$

$=2\left(1-\frac{1}{n+1}\right)=\frac{2n}{n+1}$

18. $\sum_{k=1}^{n} \dfrac{1}{k^2+k}$ 의 값은?

① $\dfrac{1}{n+1}$ ② $\dfrac{n}{n+1}$ ③ $\dfrac{2n}{n+1}$
④ $\dfrac{2n}{2n+1}$ ⑤ $\dfrac{2n}{2n+3}$

해설

(주어진 식)$= \sum_{k=1}^{n} \left(\dfrac{1}{k} - \dfrac{1}{k+1}\right)$

$= \left(\dfrac{1}{1} - \dfrac{1}{2}\right) + \left(\dfrac{1}{2} - \dfrac{1}{3}\right) + \left(\dfrac{1}{3} - \dfrac{1}{4}\right) + \cdots + \left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

$= 1 - \dfrac{1}{n+1} = \dfrac{n}{n+1}$

19. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$ 의 값은?

① $\frac{n(3n+5)}{4(n+1)(n+2)}$　② $\frac{n(3n+5)}{4(2n+1)(n+2)}$
③ $\frac{n(3n+5)}{(n+1)(n+2)}$　④ $\frac{n(3n+4)}{4(n+1)(n+2)}$
⑤ $\frac{n(3n+4)}{2(n+1)(n+2)}$

해설

$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$

$=\sum_{k=1}^{n}\frac{1}{k(k+2)}$

$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\}$

$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$

$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\frac{n(3n+5)}{4(n+1)(n+2)}$

20. $\sum_{k=1}^{n} \frac{1}{4k^2-1}$ 의 값은?

① $\frac{1}{n+1}$　② $\frac{n}{n+1}$　③ $\frac{2n}{n+1}$　④ $\frac{n}{2n+1}$　⑤ $\frac{2n}{2n+3}$

해설

(주어진 식) $= \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)$

$\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)\right\}$

$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right\}$

$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$