

1. $\sum_{i=1}^{100} x_i = 4, \sum_{i=1}^{100} y_i = 6$ 일때, $\sum_{k=1}^{100} (3x_k - 2y_k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 0

해설

$$\begin{aligned}\sum_{k=1}^{100} (3x_k - 2y_k) &= 3 \sum_{k=1}^{100} x_k - 2 \sum_{k=1}^{100} y_k \\ &= 3 \sum_{i=1}^{100} x_i - 2 \sum_{i=1}^{100} y_i = 3 \cdot 4 - 2 \cdot 6 = 0\end{aligned}$$

2. $\sum_{k=1}^5 a_k = 5$, $\sum_{k=1}^5 b_k = 7$ 일 때, $\sum_{k=1}^5 (3a_k + 2b_k)$ 의 값은?

① 21

② 22

③ 23

④ 24

⑤ 29

해설

$$\begin{aligned}\sum_{k=1}^5 (3a_k + 2b_k) &= \sum_{k=1}^5 3a_k + \sum_{k=1}^5 2b_k \\ &= 3 \sum_{k=1}^5 a_k + 2 \sum_{k=1}^5 b_k \\ &= 3 \times 5 + 2 \times 7 = 15 + 14 = 29\end{aligned}$$

3. 다음 중 옳은 것은?

① $1 + 4 + 7 + \cdots + (3n - 5) = \sum_{k=1}^n (3k - 5)$

② $2 + 4 + 6 + \cdots + 2(n + 1) = \sum_{k=1}^n 2(k + 1)$

③ $3 + 5 + 7 + \cdots + (2n - 1) = \sum_{k=1}^n (2k + 1)$

④ $4 + 5 + 6 + \cdots + (n + 3) = \sum_{k=1}^n (k + 3)$

⑤ $3 + 4 + 5 + \cdots + n = \sum_{k=1}^n k$

해설

① $1 + 4 + 7 + \cdots + (3n - 5) = \sum_{k=1}^{n-1} (3k - 2)$

② $2 + 4 + 6 + \cdots + 2(n + 1) = \sum_{k=1}^{n+1} 2n$

③ $3 + 5 + 7 + \cdots + (2n - 1) = \sum_{k=1}^{n-1} (2k + 1)$

⑤ $3 + 4 + 5 + \cdots + n = \sum_{k=1}^{n-2} (k + 2)$

4. $\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 470

해설

$$\begin{aligned}\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2 &= (\sum_{k=1}^{15} k^2 - \sum_{k=1}^{10} k^2) - \sum_{k=1}^{10} k^2 \\ &= \sum_{k=1}^{15} k^2 - 2 \sum_{k=1}^{10} k^2 \\ &= \frac{15 \cdot 16 \cdot 31}{6} - 2 \cdot \frac{10 \cdot 11 \cdot 21}{6} = 470\end{aligned}$$

5. $\sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\}$ 의 값은?

① 385

② 550

③ 1100

④ 1150

⑤ 1200

해설

$$\begin{aligned} & \sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\} \\ &= \sum_{j=1}^{10} \left\{ 3j + \frac{j(j+1)}{2} \right\} \\ &= \sum_{j=1}^{10} \left(\frac{j^2 + 7j}{2} \right) \\ &= \frac{1}{2} \left(\sum_{j=1}^{10} j^2 + 7 \sum_{j=1}^{10} j \right) \\ &= \frac{1}{2} \left(\frac{10 \cdot 11 \cdot 21}{6} + 7 \times \frac{10 \cdot 11}{2} \right) \\ &= \frac{1}{2} (385 + 385) = 385 \end{aligned}$$

6. $\sum_{k=1}^n a_k = n^2 + 2n$ 일 때, $\sum_{k=1}^3 (a_k + 1)^2 - \sum_{k=1}^3 (a_k - 1)^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 60

해설

$$\begin{aligned} & \sum_{k=1}^3 (a_k + 1)^2 - \sum_{k=1}^3 (a_k - 1)^2 \\ &= \sum_{k=1}^3 (a_k + 2a_k + 1) - \sum_{k=1}^3 (a_k^2 - 2a_k + 1) \\ &= 4 \sum_{k=1}^3 a_k = 4(3^2 + 2 \times 3) = 60 \end{aligned}$$

7. $\sum_{l=1}^n (\sum_{k=1}^l k) = 364$ 를 만족하는 n 의 값은?

① 10

② 11

③ 12

④ 13

⑤ 14

해설

$$\sum_{l=1}^n (\sum_{k=1}^l k) = \sum_{l=1}^n \left\{ \frac{l(l+1)}{2} \right\} = \frac{1}{2} \sum_{l=1}^n (l^2 + l)$$

$$= \frac{1}{2} \left\{ \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right\}$$

$$= \frac{1}{2} \times \frac{n(n+1)(n+2)}{3}$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$= 364 = 2^2 \times 7 \times 13$$

$$\therefore n(n+1)(n+2) = 6 \times 2^2 \times 7 \times 13 = 12 \times 13 \times 14$$

따라서 $n = 12$

8. 수열 $2 \cdot 3, 3 \cdot 5, 4 \cdot 7, 5 \cdot 9, \dots$ 의 제 n 항까지의 합은?

① $4n^2 + 15n + 17$

② $n(4n^2 + 15n + 17)$

③ $\frac{4n^2 + 15n + 17}{3}$

④ $\frac{n(4n^2 + 15n + 17)}{3}$

⑤ $\frac{n(4n^2 + 15n + 17)}{6}$

해설

$$a_k = (k+1)(2k+1) = 2k^2 + 3k + 1 \text{ 이므로}$$

$$S_n = \sum_{k=1}^n (2k^2 + 3k + 1)$$

$$= 2 \sum_{k=1}^n k^2 + 3 \sum_{k=1}^n k + \sum_{k=1}^n 1$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + 3 \cdot \frac{n(n+1)}{2} + n$$

$$= \frac{n(4n^2 + 15n + 17)}{6}$$

9. 다음 수열의 첫째항부터 제 10항까지의 합은?

$$1 \cdot 1 \cdot 3, 2 \cdot 3 \cdot 5, 3 \cdot 5 \cdot 7, 4 \cdot 7 \cdot 9, \dots$$

① 10050

② 11000

③ 11055

④ 12045

⑤ 12100

해설

주어진 수열의 일반항은 $n(2n-1)(2n+1) = 4n^3 - n$ 이므로
첫째항부터 제 10항까지의 합은

$$\begin{aligned}\sum_{k=1}^{10} (4k^3 - k) &= 4 \cdot \left(\frac{10 \times 11}{2} \right)^2 - \frac{10 \times 11}{2} \\ &= 12100 - 55 = 12045\end{aligned}$$

10. $1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1$ 의 값은?

① 1310

② 1320

③ 1330

④ 1340

⑤ 1350

해설

$$\begin{aligned} & 1 \cdot 19 + 2 \cdot 18 + 3 \cdot 17 + \cdots 19 \cdot 1 \\ &= 1 \cdot (20 - 1) + 2 \cdot (20 - 2) + 3 \cdot (20 - 3) + \cdots + 19 \cdot (20 - 19) \\ &= \sum_{k=1}^{19} k(20 - k) = \sum_{k=1}^{19} (20k - k^2) \\ &= 20 \times \frac{19 \cdot 20}{2} - \frac{19 \cdot 20 \cdot 39}{6} \\ &= 190(20 - 13) = 1330 \end{aligned}$$

11. 2^n 을 3으로 나눈 나머지를 a_n 이라 할 때, $\sum_{k=1}^{12} a_k$ 의 값은?

① 16

② 17

③ 18

④ 19

⑤ 20

해설

$a_1 = 2^1$ 을 3으로 나눈 나머지 $a_1 = 2$

$a_2 = 2^2$ 을 3으로 나눈 나머지 $a_2 = 1$

$a_3 = 2^3$ 을 3으로 나눈 나머지 $a_3 = 2$

$a_4 = 2^4$ 을 3으로 나눈 나머지 $a_4 = 1$

$a_5 = 2^5$ 을 3으로 나눈 나머지 $a_5 = 2$

즉 a_n 은 n 이 홀수일 때는 2

n 이 짝수일 때는 1

$$\sum_{k=1}^{12} a_k = 6 \cdot (2 + 1) = 6 \cdot 3 = 18$$

12. 수열 $\{a_n\}$ 이 $\sum_{k=1}^n a_{2k-1} = n^2$, $\sum_{k=1}^n a_{2k} = 2^n$ 을 만족할 때, $a_9 + a_{10}$ 의 값은?

① 20

② 22

③ 25

④ 27

⑤ 30

해설

$n \geq 2$ 일 때,

$$a_{2n-1} = \sum_{k=1}^n a_{2k-1} - \sum_{k=1}^{n-1} a_{2k-1} = n^2 - (n-1)^2 = 2n-1$$

$$\therefore a_9 = 2 \cdot 5 - 1 = 9$$

$$a_{2n} = \sum_{k=1}^n a_{2k} - \sum_{k=1}^{n-1} a_{2k} = 2^n - 2^{n-1} = 2^{n-1}$$

$$\therefore a_{10} = 2^{5-1} = 16$$

$$\therefore a_9 + a_{10} = 25$$

13. $\sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}}$ 의 값은?

① $\sqrt{n-1} - 1$

② $\sqrt{n+1} - 1$

③ $\sqrt{n+1}$

④ $\sqrt{n+1} + 1$

⑤ $\sqrt{2n+1} + 1$

해설

$$\begin{aligned}\frac{1}{\sqrt{k} + \sqrt{k+1}} &= \frac{\sqrt{k+1} - \sqrt{k}}{(\sqrt{k+1} + \sqrt{k})(\sqrt{k+1} - \sqrt{k})} \\ &= \frac{\sqrt{k+1} - \sqrt{k}}{(k+1) - k} = \sqrt{k+1} - \sqrt{k}\end{aligned}$$

따라서

$$\begin{aligned}(\text{주어진 식}) &= \sum_{k=1}^n (\sqrt{k+1} - \sqrt{k}) \\ &= (\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \cdots + (\sqrt{n+1} - \sqrt{n}) \\ &= \sqrt{n+1} - 1\end{aligned}$$

14. $\sum_{k=1}^n \frac{1}{(2k-1)(2k+1)}$ 의 값은?

① $\frac{1}{n+1}$

② $\frac{2n}{n+1}$

③ $\frac{n}{2n+1}$

④ $\frac{n}{n+2}$

⑤ $\frac{2n}{2n+1}$

해설

$$\begin{aligned}(\text{준 식}) &= \frac{1}{2} \sum_{k=1}^n \left\{ \frac{1}{2k-1} - \frac{1}{2k+1} \right\} \\&= \frac{1}{2} \cdot \left\{ \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \cdots \right\} \\&\quad + \frac{1}{2} \left\{ \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \right\} \\&= \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{n}{2n+1}\end{aligned}$$

15. 함수 $f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$ 에 대하여 $\sum_{k=1}^{20} \frac{2k+1}{f(k)}$ 의 값은?

① $\frac{40}{7}$

② $\frac{45}{8}$

③ $\frac{17}{3}$

④ $\frac{57}{10}$

⑤ $\frac{63}{11}$

해설

$$\begin{aligned} f(n) &= 1^2 + 2^2 + 3^2 + \cdots + n^2 \\ &= \sum_{k=1}^{20} k^2 = \frac{n(n+1)(2n+1)}{6} \text{ 이므로} \\ \sum_{k=1}^{20} \frac{2k+1}{f(x)} &= \sum_{k=1}^{20} \frac{2k+1}{\frac{k(k+1)(2k+1)}{6}} \\ &= \sum_{k=1}^{20} \frac{6}{k(k+1)} = 6 \sum_{k=1}^{20} \left(\frac{1}{k} - \frac{1}{k+1} \right) \\ &= 6 \left(1 - \frac{1}{21} \right) = 6 \times \frac{20}{21} = \frac{40}{7} \end{aligned}$$

16. x 에 대한 이차방정식 $x^2 + 4x - (2n - 1)(2n + 1) = 0$ 의 두 근 α_n, β_n 에 대하여 $\sum_{k=1}^{10} \left(\frac{1}{\alpha_n} + \frac{1}{\beta_n} \right)$ 의 값은?

① $\frac{11}{21}$

② $\frac{20}{21}$

③ $\frac{31}{21}$

④ $\frac{40}{21}$

⑤ $\frac{50}{21}$

해설

$$\begin{aligned}
 (\text{준 식}) &= \sum_{n=1}^{10} \frac{\alpha_n + \beta_n}{\alpha_n \cdot \beta_n} \\
 &= \sum_{n=1}^{10} \frac{-4}{-(2n-1)(2n+1)} \\
 &= 4 \sum_{n=1}^{10} \frac{1}{2} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \\
 &= 2 \sum_{n=1}^{10} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \\
 &= 2 \left(1 - \frac{1}{21} \right) = \frac{40}{21}
 \end{aligned}$$

17. $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+n}$ 의 값을 구하면?

① $\frac{n}{n+1}$

② $\frac{2n}{n+1}$

③ $\frac{3n}{n+1}$

④ $\frac{4n}{n+1}$

⑤ $\frac{5n}{n+1}$

해설

$$(\text{주어진 식}) = \sum_{k=1}^n \frac{1}{\frac{k(k+1)}{2}}$$

$$= 2 \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$= 2 \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{n} - \frac{1}{n+1} \right)$$

$$= 2 \left(1 - \frac{1}{n+1} \right) = \frac{2n}{n+1}$$

18. $\sum_{k=1}^n \frac{1}{k^2 + k}$ 의 값은?

① $\frac{1}{n+1}$

② $\frac{n}{n+1}$

③ $\frac{2n}{n+1}$

④ $\frac{2n}{2n+1}$

⑤ $\frac{2n}{2n+3}$

해설

$$\begin{aligned}(\text{주어진 식}) &= \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) \\&= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1} \right) \\&= 1 - \frac{1}{n+1} = \frac{n}{n+1}\end{aligned}$$

19. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{n(n+2)}$ 의 값은?

① $\frac{n(3n+5)}{4(n+1)(n+2)}$

② $\frac{n(3n+5)}{4(2n+1)(n+2)}$

③ $\frac{n(3n+5)}{(n+1)(n+2)}$

④ $\frac{n(3n+4)}{4(n+1)(n+2)}$

⑤ $\frac{n(3n+4)}{2(n+1)(n+2)}$

해설

$$\begin{aligned}
 & \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{n(n+2)} \\
 &= \sum_{k=1}^n \frac{1}{k(k+2)} \\
 &= \frac{1}{2} \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+2} \right) \\
 &= \frac{1}{2} \left\{ \left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) \right\} \\
 &+ \cdots + \frac{1}{2} \left\{ \left(\frac{1}{n-1} - \frac{1}{n+1} \right) + \left(\frac{1}{n} - \frac{1}{n+2} \right) \right\} \\
 &= \frac{1}{2} \left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right) \\
 &= \frac{n(3n+5)}{4(n+1)(n+2)}
 \end{aligned}$$

20. $\sum_{k=1}^n \frac{1}{4k^2 - 1}$ 의 값은?

① $\frac{1}{n+1}$

② $\frac{n}{n+1}$

③ $\frac{2n}{n+1}$

④ $\frac{n}{2n+1}$

⑤ $\frac{2n}{2n+3}$

해설

$$(\text{주어진 식}) = \frac{1}{2} \sum_{k=1}^n \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right)$$

$$\frac{1}{2} \left\{ \left(\frac{1}{1} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) \right\}$$

$$+ \cdots + \frac{1}{2} \left\{ \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \right\}$$

$$= \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{n}{2n+1}$$