1. 수열 $\{a_n\}$에 대하여 $a_1 = 1,\ a_{11} = 32$일 때, $\sum_{k=1}^{10}(a_{k+1} - a_k)$의 값은?

① 25 ② 27 ③ 29 ④ 31 ⑤ 33

해설

$\sum_{k=1}^{10}(a_{k+1} - a_k)$
$= (a_2 - a_1) + (a_3 - a_2) + (a_4 - a_3) + \cdots + (a_{11} - a_{10})$
$= a_{11} - a_1 = 32 - 1 = 31$

2. 수열 $\{a_n\}$이 $a_1 = 1$, $a_{10} = 30$을 만족할 때 $\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$의 값은?

① 26 ② 27 ③ 28 ④ 29 ⑤ 30

해설

$\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$
$= (a_2 + a_3 + \cdots + a_9 + a_{10}) -$
$(a_1 + a_2 + \cdots + a_9)$
$= -a_1 + a_{10} = -1 + 30 = 29$

3. $\sum_{k=3}^{10} k(k+2)$ 의 값은?

① 460 ② 468 ③ 478 ④ 480 ⑤ 484

해설

$\sum_{k=1}^{10} k(k+2) = \sum_{k=1}^{10} k(k+2) - \sum_{k=1}^{2} k(k+2)$

$= \sum_{k=1}^{10} (k^2+2k) - \sum_{k=1}^{2} (k^2+2k)$

$= \sum_{k=1}^{10} k^2 + 2\sum_{k=1}^{10} k - (3+8)$

$= \dfrac{10 \cdot 11 \cdot 21}{6} + 2 \cdot \dfrac{10 \cdot 11}{2} - 11$

$= 385 + 110 - 11$

$= 484$

4. $\sum_{l=1}^{10}\left\{\sum_{k=1}^{5}(k+l)\right\}$ 의 값은?

① 400　② 425　③ 450　④ 475　⑤ 500

해설

$\sum_{l=1}^{5}(k+l)=\sum_{k=1}^{5}k+\sum_{k=1}^{5}l=\sum_{k=1}^{5}k+5l$

$\therefore$ (준 식) $=\sum_{l=1}^{10}(5l+15)=5\sum_{l=1}^{10}l+150$

$=5\times 55+150=425$

5. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$ 의 값은?

① 385 ② 550 ③ 1100 ④ 1150 ⑤ 1200

해설

$\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$

$=\sum_{j=1}^{10}\left\{3j+\frac{j(j+1)}{2}\right\}$

$=\sum_{j=1}^{10}\left(\frac{j^2+7j}{2}\right)$

$=\frac{1}{2}\left(\sum_{j=1}^{10}j^2+7\cdot\sum_{j=1}^{10}j\right)$

$=\frac{1}{2}\left(\frac{10\cdot 11\cdot 12}{6}+7\times\frac{10\cdot 11}{2}\right)$

$=\frac{1}{2}(385+385)$

$=385$

6. $\sum_{k=1}^{n} a_k = 10n$, $\sum_{k=1}^{n} b_k = 5n$ 일 때, $\sum_{n=1}^{10} \left\{\sum_{k=1}^{n}(2a_k - 3b_k + 5)\right\}$ 의 값은?

① 250　② 300　③ 450　④ 550　⑤ 650

해설

$\sum_{n=1}^{10} \left\{2\sum_{k=1}^{n} a_k - 3\sum_{k=1}^{n} b_k + \sum_{k=1}^{n} 5\right\}$
$= \sum_{n=1}^{10}(2 \cdot 10n - 3 \cdot 5n + 5n)$
$= \sum_{n=1}^{10}(20n - 15n + 5n)$
$= \sum_{n=1}^{10} 10n = 10 \cdot \frac{10 \cdot 11}{2}$
$= 550$

7. 다음 수열의 합을 $\sum$기호를 써서 나타내면?

$3+6+12+\cdots+3\cdot 2^{n-1}$

① $\sum_{k=1}^{n} 3\cdot 2^{k-1}$ ② $\sum_{k=1}^{n-1} 3\cdot 2^{k-1}$ ③ $\sum_{k=1}^{n} 3\cdot 2^{k}$
④ $\sum_{k=1}^{n-1} 3\cdot 2^{k}$ ⑤ $\sum_{k=1}^{n} 3\cdot 2^{k+1}$

해설

제 k항은 $3\cdot 2^{k-1}$, 항 수는 n이므로
$3+6+9+\cdots+3\cdot 2^{n-1}=\sum_{k=1}^{n} 3\cdot 2^{k-1}$

8. 다음 식의 값은?

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

① 9　② $3\sqrt{11}-\sqrt{2}$　③ $\sqrt{99}-1$
④ $\sqrt{101}-1$　⑤ 11

해설

(준식)$=\sum_{k=1}^{99}\frac{1}{\sqrt{k}+\sqrt{k-1}}=\sum_{k=1}^{99}(\sqrt{k+1}-\sqrt{k})$

$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+\cdots+(\sqrt{100}-\sqrt{99})$

$=\sqrt{100}-1=9$

9. $\sum_{k=1}^{80}(\sqrt{k}-\sqrt{k+1})$ 의 값은?

① -5　② -7　③ -8　④ -79　⑤ -80

해설

$\sum_{k=1}^{80}(\sqrt{k}-\sqrt{k+1})$
$=\sqrt{1}-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\cdots+\sqrt{80}-\sqrt{81}$
$=\sqrt{1}-\sqrt{81}$
$=1-9=-8$

10. $\sum_{k=1}^{49} \frac{1}{\sqrt{k}+\sqrt{k+1}} = a\sqrt{2}+b$일 때, $a+b$의 값은?

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

해설

$\sum_{k=1}^{49} \frac{1}{\sqrt{k}+\sqrt{k+1}}$

$= \sum_{k=1}^{49} \frac{\sqrt{k}-\sqrt{k+1}}{(\sqrt{k}+\sqrt{k+1})(\sqrt{k}-\sqrt{k+1})}$

$= \sum_{k=1}^{49} (\sqrt{k}-\sqrt{k+1})$

$= -\left\{(\sqrt{1}-\sqrt{2})+(\sqrt{2}-\sqrt{3})+\cdots\right.$

$\left.+(\sqrt{49}-\sqrt{50})\right\}$

$= -(1-\sqrt{50}) = 5\sqrt{2}-1$

따라서, $a=5,\ b=-1$에서 $a+b=4$

11. $\sum_{k=1}^{200} \frac{1}{k(k+1)}$ 의 값은?

① $\frac{101}{100}$ ② $\frac{100}{101}$ ③ $\frac{200}{201}$ ④ $\frac{110}{101}$ ⑤ $\frac{201}{200}$

해설

$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ 이므로

$\sum_{k=1}^{200} \frac{1}{k(k+1)} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{199} - \frac{1}{200}\right) + \left(\frac{1}{200} - \frac{1}{201}\right)$

$= \frac{1}{1} - \frac{1}{201} = \frac{200}{201}$

12. $\sum_{k=1}^{n} \frac{1}{4k^2-1}$ 의 값은?

① $\frac{1}{n+1}$　② $\frac{n}{n+1}$　③ $\frac{2n}{n+1}$

④ $\frac{n}{2n+1}$　⑤ $\frac{2n}{2n+3}$

해설

(주어진 식) $= \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)$

$\frac{1}{2}\left\{\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right)\right\}$

$+ \cdots + \frac{1}{2}\left\{\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)\right\}$

$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$

13. $\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1) = 62$를 만족하는 자연수 n의 값을 구하여라.

▶ 답 :

▷ 정답 : 7

해설

$\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1)$

$=\sum_{k=1}^{n}(k^2+1)-\left\{\sum_{k=1}^{n}(k^2-1)-(n^2-1)\right\}$

$=\sum_{k=1}^{n}\left\{(k^2+1)-(k^2-1)\right\}+(n^2-1)$

$=\sum_{k=1}^{n}2+(n^2-1)=n^2+2n-1=62$

이것을 정리하여 인수분해하면 $(n+9)(n-7)=0$

따라서 $n=-9$ 또는 $n=7$

그런데 $n>0$이므로 $n=7$

14. 수열 $\{a_n\}$에 대하여 $\sum_{k=1}^{n}(a_{2k-1}+a_{2k})=8n^2+10n$일 때, $\sum_{k=1}^{10}a_k$의 값을 구하여라.

▶ 답 :

▷ 정답 : 250

해설

$\sum_{k=1}^{10}a_k=a_1+a_2+a_3+\cdot+a_{10}$
$=(a_1+a_2)+(a_3+a_4)+\cdots+(a_9+a_{10})$
$=\sum_{k=1}^{5}(a_{2k-1}+a_{2k})$
$=8\times5^2+10\times5=250$

15. 다음 등식이 성립하도록 하는 c의 값을 구하여라.

$$\sum_{k=11}^{100}(k-2)^2=\sum_{k=11}^{100}k^2-4\sum_{k=11}^{100}k+c$$

▶ 답 :

▷ 정답 : 360

해설

$\sum_{k=11}^{100}(k-2)^2=\sum_{k=11}^{100}(k^2-4k+4)$
$=\sum_{k=11}^{100}-4\sum_{k=11}^{100}k+\sum_{k=11}^{100}4$
$\therefore\ c=\sum_{k=11}^{100}4=4+4+\cdots+4=4\times 90=360$

16. $S=\sum_{k=1}^{10}k+\sum_{k=2}^{10}k+\sum_{k=3}^{10}k+\cdots+\sum_{k=9}^{10}k+\sum_{k=10}^{10}k$일 때, $\frac{1}{5}S$의 값을 구하여라.

▶ 답 :

▷ 정답 : 77

해설

$S=\sum_{k=1}^{10}k+\sum_{k=2}^{10}k+\sum_{k=3}^{10}k+\cdots+\sum_{k=9}^{10}k+\sum_{k=10}^{10}k$

$=1+2+3+4+\cdots+10$

$+2+3+4+\cdots+10$

$3+4+\cdots+10$

$\vdots$

$+10$

$=1+2^2+3^2+4^2+\cdots+10^2$

$=\frac{10\times11\times21}{6}=385$

$\therefore\ \frac{1}{5}S=77$

17. $\sum_{l=1}^{n}\left(\sum_{k=1}^{l} 12k\right) = 1008$을 만족시키는 n의 값은?

① 5 ② 6 ③ 7 ④ 8 ⑤ 9

해설

$\sum_{l=1}^{n}\left(\sum_{k=1}^{l} 12k\right)$

$= \sum_{l=1}^{n} 12 \cdot \left\{\frac{l(l+1)}{2}\right\} = 6\left(\sum_{l=1}^{n} l^2 + \sum_{l=1}^{n} l\right)$

$= 6\left\{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right\}$

$= n(n+1)(2n+4) = 2n(n+1)(n+2)$

즉, $2n(n+1)(n+2) = 1008$이므로

$n(n+1)(2n+4) = 7 \cdot 8 \cdot 9 = 504$

$\therefore\ n = 7$

18. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 20 \cdot 21$ 의 값은?

① 2200 ② 2640 ③ 2860 ④ 3020 ⑤ 3080

해설

$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 20 \cdot 21 = \sum_{k=1}^{20} k(k+1)$

$= \sum_{k=1}^{20} k^2 + \sum_{k=1}^{20} k = \frac{20 \cdot 21 \cdot 41}{6} + \frac{20 \cdot 21}{2}$

$= 2870 + 210 = 3080$

19. 다음을 계산하여라.

$$1\cdot 1+2\cdot 4+3\cdot 7+\cdots+10\cdot 28$$

▶ 답 :

▷ 정답 : 1045

해설

$1\cdot 1+2\cdot 4+3\cdot 7+\cdots+10\cdot 28$
$=\sum_{k=1}^{10} k\cdot(3k-2)$
$=\sum_{k=1}^{10}(3k^2-2k)$
$=3\sum_{k=1}^{10} k^2-2\sum_{k=1}^{10} k$
$=3\cdot\dfrac{10\cdot 11\cdot 21}{6}-2\cdot\dfrac{10\cdot 11}{2}$
$=1155-110$
$=1045$

20. 다음 수열의 첫째항부터 제 10항까지의 합은?

$1 \cdot 1 \cdot 3,\ 2 \cdot 3 \cdot 5,\ 3 \cdot 5 \cdot 7,\ 4 \cdot 7 \cdot 9,\ \cdots$

① 10050 ② 11000 ③ 11055

④ 12045 ⑤ 12100

해설

주어진 수열의 일반항은 $n(2n-1)(2n+1) = 4n^3 - n$ 이므로
첫째항부터 제 10항까지의 합은

$\sum_{k=1}^{10}(4k^3 - k) = 4 \cdot \left(\frac{10 \times 11}{2}\right)^2 - \frac{10 \times 11}{2}$

$= 12100 - 55 = 12045$

21. n개의 수 $1 \cdot 2n,\ 2 \cdot (2n-1),\ 3 \cdot (2n-2), \cdots,\ n(n+1)$ 의 합은?

① $\dfrac{n^2(n+1)}{2}$　② $\dfrac{n(n+1)^2}{2}$

③ $\dfrac{(n+1)(2n+1)}{6}$　④ $\dfrac{(n+1)(2n+1)}{3}$

⑤ $n(n+1)(2n+1)$

해설

주어진 수열의 제 k항은

$k\left\{2n-(k-1)\right\} = k(2n-k+1)$

$= -k^2 + (2n+1)k$

이므로 구하는 합은

$\sum_{k=1}^{n} k\left\{2n-(k-1)\right\}$

$= -\sum_{k=1}^{n} k^2 + (2n+1)\sum_{k=1}^{n} k$

$= -\dfrac{n(n+1)(2n+1)}{6} + (2n+1) \times \dfrac{n(n+1)}{2}$

$= \dfrac{n(n+1)(2n+1)}{3}$

22. $\sum_{k=1}^{n} a_k = 2n^2 - n$ 일 때, $\sum_{k=1}^{5}(2k+1)a_k$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 395

해설

$a_n = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n-1} a_k$
$= (2n^2 - n) - \{2(n-1)^2 - (n-1)\}$
$= 4n - 3(n = 2,\ 3,\ 4,\ \cdots)$
$n = 1$ 일 때, $a_1 = 2 \cdot 1^2 - 1 = 1$
따라서 $a_n = 4n - 3(n = 1,\ 2,\ 3,\ \cdots)$ 이므로
$\sum_{k=1}^{5}(2k+2)a_k = \sum_{k=1}^{5}(2k+1)(4k-3)$
$= \sum_{k=1}^{5}(8k^2 - 2k - 3)$
$= 8 \cdot \frac{5 \cdot 6 \cdot 11}{6} - 2 \cdot \frac{5 \cdot 6}{2} - 3 \cdot 5$
$= 440 - 30 - 15 = 395$

23. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$ 의 값은?

① $\frac{n(3n+5)}{4(n+1)(n+2)}$ ② $\frac{n(3n+5)}{4(2n+1)(n+2)}$

③ $\frac{n(3n+5)}{(n+1)(n+2)}$ ④ $\frac{n(3n+4)}{4(n+1)(n+2)}$

⑤ $\frac{n(3n+4)}{2(n+1)(n+2)}$

해설

$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$

$=\sum_{k=1}^{n}\frac{1}{k(k+2)}$

$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$

$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\}$

$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$

$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\frac{n(3n+5)}{4(n+1)(n+2)}$

24. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ 의 값은?

① $\frac{1}{6}$ ② $\frac{1}{3}$ ③ $\frac{1}{2}$ ④ $\frac{2}{3}$ ⑤ $\frac{5}{6}$

해설

(준 식)$=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{3}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$

$=1-\frac{1}{6}=\frac{5}{6}$

25. $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)}$ 의 값은?

① $\frac{1}{n+1}$ ② $\frac{2n}{n+1}$ ③ $\frac{n}{2n+1}$
④ $\frac{n}{n+2}$ ⑤ $\frac{2n}{2n+1}$

해설

준 식) $= \frac{1}{2}\sum_{k=1}^{n}\left\{\frac{1}{2k-1} - \frac{1}{2k+1}\right\}$

$= \frac{1}{2} \cdot \left\{\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)\right\} + \cdots +$

$\frac{1}{2}\left\{\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right\}$

$= \frac{1}{2}\left(1-\frac{1}{2n+1}\right)$

$= \frac{n}{2n+1}$

26. 첫째항부터 제 n항까지의 합이 $S_n = n^3 - n$인 수열 $\{a_n\}$에서 $\frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_{20}}$의 값은?

① $\frac{17}{19}$ ② $\frac{17}{30}$ ③ $\frac{19}{40}$ ④ $\frac{17}{50}$ ⑤ $\frac{19}{60}$

해설

$a_n = S_n - S_{n-1} = (n^3 - n) - \left\{(n-1)^3 - (n-1)\right\} = 3n(n-1)(n \geq 2)$

$\therefore \frac{1}{a_n} = \frac{1}{3n(n-1)} = \frac{1}{3}\left(\frac{1}{n-1} - \frac{1}{n}\right) (n \geq 2)$

$\therefore \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_{20}}$

$= \frac{1}{3}\left\{\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{19} - \frac{1}{20}\right)\right\}$

$= \frac{1}{3}\left(1 - \frac{1}{20}\right) = \frac{19}{60}$

27. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+3+\cdots+10}$ 의 값은?

① $\frac{9}{10}$ ② $\frac{11}{10}$ ③ $\frac{10}{11}$ ④ $\frac{20}{11}$ ⑤ $\frac{11}{20}$

해설

$\frac{1}{1+2+\cdots+n}=\frac{1}{\frac{n(n+1)}{2}}=\frac{2}{n(n+1)}$

$\therefore \sum_{k=1}^{10}\frac{2}{k(k+1)}=2\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$=2\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right)\right\}$

$=2\left(1-\frac{1}{11}\right)=\frac{20}{11}$

28. $\sum_{k=1}^{n} a_k = n^2 + 3n$ 일 때, $\sum_{k=1}^{10} \frac{1}{a_k a_{k+1}}$ 의 값은?

① $\frac{1}{24}$　② $\frac{1}{48}$　③ $\frac{5}{16}$　④ $\frac{5}{24}$　⑤ $\frac{5}{48}$

해설

$a_n = S_n - S_{n-1}$

$= n^2 + 3n - \{(n-1)^2 + 3(n-1)\} = 2n + 2 (n \geq 2)$

$a_1 = 1 + 3 = 2 + 2 = 4$ 이므로, $a_n = 2n + 2 (n \geq 1)$

$\sum_{k=1}^{10} \frac{1}{a_k a_{k+1}} = \sum_{k=1}^{10} \frac{1}{(2k+2)(2k+4)}$

$= \frac{1}{4} \sum_{k=1}^{10} \left(\frac{1}{k+1} - \frac{1}{k+2} \right)$

$= \frac{1}{4} \left\{ \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{11} - \frac{1}{12} \right) \right\}$

$= \frac{1}{4} \left(\frac{1}{2} - \frac{1}{12} \right) = \frac{5}{24}$

29. $\sum_{k=1}^{10}\left\{\sum_{m=1}^{n}(k-2)\cdot 2^{m-1}\right\}$ 을 n에 관한 식으로 나타내면?

① $60(2^n-1)$　② $35(2^n-1)$　③ $20(2^n+1)$

④ $20(2^n-1)$　⑤ $16(2^n-1)$

해설

$$\sum_{k=1}^{10}\left\{\sum_{m=1}^{n}(k-2)\cdot 2^{m-1}\right\}$$

$$=\sum_{k=1}^{10}\left\{\frac{(k-2)(2^n-1)}{2-1}\right\}$$

$$=(2^n-1)\sum_{k=1}^{10}(k-2)$$

$$=(2^n-1)\left(\frac{10\times 11}{2}-20\right)=35(2^n-1)$$

30. x에 대한 이차방정식 $x^2+4x-(2n-1)(2n+1)=0$의 두근 α_n, β_n에 대하여 $\sum_{n=1}^{10}\left(\frac{1}{\alpha_n}+\frac{1}{\beta_n}\right)$의 값은?

① $\frac{11}{21}$ ② $\frac{20}{21}$ ③ $\frac{31}{21}$ ④ $\frac{40}{21}$ ⑤ $\frac{50}{21}$

해설

$\alpha_n+\beta_n=-4$

$\alpha_n\beta_n=-(2n-1)(2n+1)$

$\frac{1}{\alpha_n}+\frac{1}{\beta_n}=\frac{\alpha_n+\beta_n}{\alpha_n\beta_n}=\frac{-4}{-(2n-1)(2n+1)}$

$\sum_{k=1}^{10}\left(\frac{1}{\alpha_n}+\frac{1}{\beta_n}\right)=\sum_{k=1}^{10}\frac{\alpha_n+\beta_n}{\alpha_n\beta_n}$

$=\sum_{k=1}^{10}\frac{4}{(2n-1)(2n+1)}$

$=4\sum_{k=1}^{10}\frac{1}{(2n-1)(2n+1)}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$=\frac{4}{2}\sum_{k=1}^{10}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$

$=2\cdot\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\cdots+\frac{1}{19}-\frac{1}{21}\right)$

$=2\left(1-\frac{1}{21}\right)=\frac{40}{21}$