

1. $\sum_{k=1}^5 (2k-1) + \sum_{k=6}^{10} (2k-1)$ 의 값은?

① 70

② 80

③ 90

④ 100

⑤ 110

해설

$$\begin{aligned} & \sum_{k=1}^5 (2k-1) + \sum_{k=6}^{10} (2k-1) \\ &= \sum_{k=1}^{10} (2k-1) = 2 \cdot \sum_{k=1}^{10} k - \sum_{k=1}^{10} 1 \\ &= 2 \cdot \frac{10 \cdot 11}{2} - 10 \\ &= 110 - 10 = 100 \end{aligned}$$

2. $\sum_{k=1}^{10} (11 - k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 55

해설

$$\begin{aligned}\sum_{k=1}^{10} (11 - k) &= 10 + 9 + 8 + \cdots + 2 + 1 \\ &= \sum_{k=1}^{10} k = \frac{10 \cdot 11}{2} = 55\end{aligned}$$

3. 다음 중 옳은 것은?

① $1 + 4 + 7 + \cdots + (3n - 5) = \sum_{k=1}^n (3k - 5)$

② $2 + 4 + 6 + \cdots + 2(n + 1) = \sum_{k=1}^n 2(k + 1)$

③ $3 + 5 + 7 + \cdots + (2n - 1) = \sum_{k=1}^n (2k + 1)$

④ $4 + 5 + 6 + \cdots + (n + 3) = \sum_{k=1}^n (k + 3)$

⑤ $3 + 4 + 5 + \cdots + n = \sum_{k=1}^n k$

해설

① $1 + 4 + 7 + \cdots + (3n - 5) = \sum_{k=1}^{n-1} (3k - 2)$

② $2 + 4 + 6 + \cdots + 2(n + 1) = \sum_{k=1}^{n+1} 2n$

③ $3 + 5 + 7 + \cdots + (2n - 1) = \sum_{k=1}^{n-1} (2k + 1)$

⑤ $3 + 4 + 5 + \cdots + n = \sum_{k=1}^{n-2} (k + 2)$

4. $\sum_{k=3}^{10} k(k+2)$ 의 값은?

① 460

② 468

③ 478

④ 480

⑤ 484

해설

$$\begin{aligned}\sum_{k=1}^{10} k(k+2) &= \sum_{k=1}^{10} k(k+2) - \sum_{k=1}^2 k(k+2) \\ &= \sum_{k=1}^{10} (k^2 + 2k) - \sum_{k=1}^2 (k^2 + 2k) \\ &= \sum_{k=1}^{10} k^2 + 2 \sum_{k=1}^{10} k - (3 + 8) \\ &= \frac{10 \cdot 11 \cdot 21}{6} + 2 \cdot \frac{10 \cdot 11}{2} - 11 \\ &= 385 + 110 - 11 \\ &= 484\end{aligned}$$

5. $\sum_{k=1}^n a_k = A$, $\sum_{k=1}^n b_k = B$ 일 때, 다음 중 옳지 않은 것은?

① $\sum_{k=1}^n (a_k + b_k) = A + B$

② $\sum_{k=1}^n (a_k - b_k) = A - B$

③ $\sum_{k=1}^n ca_k = cA$ (단, c 는 상수)

④ $\sum_{k=2}^{n+1} b_{k-1} = B - 1$

⑤ $\sum_{k=1}^n (a_k + c) = A + cn$ (단, c 는 상수)

해설

$\sum_{k=2}^{n+1} b_{k-1} = \sum_{k=1}^n b_k = B$
따라서, ④가 옳지 않다.

6. $a_1 + a_3 + a_5 + \cdots + a_{99}$ 를 $\sum$ 를 이용하여 나타내면?

- ① $\sum_{k=1}^{99} a_k$
- ② $\sum_{k=1}^{99} a_{2k-1}$
- ③ $\sum_{k=1}^{99} a_{2k+1}$
- ④ $\sum_{k=1}^{50} a_k$
- ⑤ $\sum_{k=1}^{50} a_{2k-1}$

해설

- ① $\sum_{k=1}^{99} a_k = a_1 + a_2 + a_3 + \cdots + a_{99}$
- ② $\sum_{k=1}^{99} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{197}$
- ③ $\sum_{k=1}^{99} a_{2k+1} = a_3 + a_5 + a_7 + \cdots + a_{199}$
- ④ $\sum_{k=1}^{50} a_k = a_1 + a_2 + a_3 + \cdots + a_{50}$
- ⑤ $\sum_{k=1}^{50} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{99}$

7. $\sum_{k=1}^5 a_k = 20$, $\sum_{k=1}^5 b_k = 5$ 일 때, $\sum_{k=1}^5 (2a_k - b_k - 1)$ 의 값은?

① 15

② 20

③ 25

④ 30

⑤ 35

해설

(주어진 식)

$$= 2 \sum_{k=1}^5 a_k - \sum_{k=1}^5 b_k - \sum_{k=1}^5 1$$

$$= 2 \cdot 20 - 5 - 5$$

$$= 30$$

8. $\sum_{i=1}^{100} x_i = 4$, $\sum_{i=1}^{100} y_i = 6$ 일때, $\sum_{k=1}^{100} (3x_k - 2y_k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 0

해설

$$\begin{aligned}\sum_{k=1}^{100} (3x_k - 2y_k) &= 3 \sum_{k=1}^{100} x_k - 2 \sum_{k=1}^{100} y_k \\ &= 3 \sum_{i=1}^{100} x_i - 2 \sum_{i=1}^{100} y_i = 3 \cdot 4 - 2 \cdot 6 = 0\end{aligned}$$

9. 다음 식의 값은?

$$\sum_{k=1}^{10}(k^2+k)-\sum_{k=4}^{10}(k^2+k)$$

- ① 14 ② 16 ③ 18 ④ 20 ⑤ 22

해설

$$(\text{준 식})=\sum_{k=1}^3(k^2+k)=(1^2+1)+(2^2+2)+(3^2+3)=20$$

10. $\sum_{k=1}^{10} a_k = 3$, $\sum_{k=1}^{10} b_k = 5$ 일 때, $\sum_{k=1}^{10} (a_k + 2b_k - 1)$ 의 값은?

① 1

② 2

③ 3

④ 4

⑤ 5

해설

$$\begin{aligned}\sum_{k=1}^{10} (a_k + 2b_k - 1) &= \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} 2b_k - \sum_{k=1}^{10} 1 \\ &= \sum_{k=1}^{10} a_k + 2 \sum_{k=1}^{10} b_k - \sum_{k=1}^{10} 1 \\ &= 3 + 2 \times 5 - 10 = 3\end{aligned}$$

11. 수열 $\{a_n\}$ 이 $a_1 = 1$, $a_{10} = 30$ 을 만족할 때 $\sum_{k=1}^9 a_{k+1} - \sum_{k=2}^{10} a_{k-1}$ 의 값은?

① 26

② 27

③ 28

④ 29

⑤ 30

해설

$$\begin{aligned} & \sum_{k=1}^9 a_{k+1} - \sum_{k=2}^{10} a_{k-1} \\ &= (a_2 + a_3 + \cdots + a_9 + a_{10}) - \\ & \quad (a_1 + a_2 + \cdots + a_9) \\ &= -a_1 + a_{10} = -1 + 30 = 29 \end{aligned}$$

12. $\sum_{k=1}^5 a_k = 5$, $\sum_{k=1}^5 b_k = 7$ 일 때, $\sum_{k=1}^5 (3a_k + 2b_k)$ 의 값은?

① 21

② 22

③ 23

④ 24

⑤ 29

해설

$$\begin{aligned}\sum_{k=1}^5 (3a_k + 2b_k) &= \sum_{k=1}^5 3a_k + \sum_{k=1}^5 2b_k \\ &= 3 \sum_{k=1}^5 a_k + 2 \sum_{k=1}^5 b_k \\ &= 3 \times 5 + 2 \times 7 = 15 + 14 = 29\end{aligned}$$

13. 수열 $\{a_n\}$ 에 대하여 $a_1 = 1$, $a_{11} = 32$ 일 때, $\sum_{k=1}^{10} (a_{k+1} - a_k)$ 의 값은?

① 25

② 27

③ 29

④ 31

⑤ 33

해설

$$\begin{aligned} & \sum_{k=1}^{10} (a_{k+1} - a_k) \\ &= (a_2 - a_1) + (a_3 - a_2) + (a_4 - a_3) + \cdots + (a_{11} - a_{10}) \\ &= a_{11} - a_1 = 32 - 1 = 31 \end{aligned}$$

14. 다음 수열의 합을 $\sum$ 기호를 써서 나타내면?

$3 + 6 + 12 + \cdots + 3 \cdot 2^{n-1}$

- ① $\sum_{k=1}^n 3 \cdot 2^{k-1}$

② $\sum_{k=1}^{n-1} 3 \cdot 2^{k-1}$

③ $\sum_{k=1}^n 3 \cdot 2^k$

④ $\sum_{k=1}^{n-1} 3 \cdot 2^k$

⑤ $\sum_{k=1}^n 3 \cdot 2^{k+1}$

해설

제 k 항은 $3 \cdot 2^{k-1}$, 항 수는 n 이므로
 $3 + 6 + 9 + \cdots + 3 \cdot 2^{n-1} = \sum_{k=1}^n 3 \cdot 2^{k-1}$

15. $\sum_{k=1}^{10} (a_k + 1)^2 = 100$, $\sum_{k=1}^{10} (a_k + 2)^2 = 200$ 일 때, $\sum_{k=1}^{10} a_k$ 의 값은?

- ① 35 ② 40 ③ 45 ④ 50 ⑤ 55

해설

$$\begin{aligned} \sum_{k=1}^{10} (a_k^2 + 2a_k + 1) &= 100 \cdots \textcircled{A} \\ \sum_{k=1}^{10} (a_k^2 + 4a_k + 4) &= 200 \cdots \textcircled{B} \\ \textcircled{B} - \textcircled{A} \text{을 하면 } \sum_{k=1}^{10} (2a_k + 3) &= 100 \\ 2 \sum_{k=1}^{10} a_k + 30 &= 100, \quad 2 \sum_{k=1}^{10} a_k = 70 \\ \therefore \sum_{k=1}^{10} a_k &= 35 \end{aligned}$$

16. 다음을 계산하여라.

$1 \cdot 1 + 2 \cdot 4 + 3 \cdot 7 + \cdots + 10 \cdot 28$

▶ 답 :

▷ 정답 : 1045

해설

$$\begin{aligned} &1 \cdot 1 + 2 \cdot 4 + 3 \cdot 7 + \cdots + 10 \cdot 28 \\ &= \sum_{k=1}^{10} k \cdot (3k - 2) \\ &= \sum_{k=1}^{10} (3k^2 - 2k) \\ &= 3 \sum_{k=1}^{10} k^2 - 2 \sum_{k=1}^{10} k \\ &= 3 \cdot \frac{10 \cdot 11 \cdot 21}{6} - 2 \cdot \frac{10 \cdot 11}{2} \\ &= 1155 - 110 \\ &= 1045 \end{aligned}$$

17. $1 \cdot 20 + 2 \cdot 19 + 3 \cdot 18 + \cdots + 20 \cdot 1$ 의 값은?

① 442

② 882

③ 1540

④ 3080

⑤ 3528

해설

$$\begin{aligned}(\text{준식}) &= \sum_{k=1}^{20} k(21-k) \\&= 21 \sum_{k=1}^{20} k - \sum_{k=1}^{20} k^2 \\&= 21 \cdot \frac{20 \cdot 21}{2} - \frac{20 \cdot 21 \cdot 41}{6} = 1540\end{aligned}$$

18. 다음 보기 중 옳은 것을 모두 고른 것은?

보기

- ㉠ $3 + 9 + \cdots + 3^{n-1} = \sum_{k=1}^{n-1} 3^{k-1}$
- ㉡ $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + n \cdot 1 = \sum_{k=1}^n k(n-k)$
- ㉢ $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 10 \cdot 2^9 = \sum_{k=1}^{10} k \cdot 2^{k-1}$

- ① ㉠
- ② ㉡
- ③ ㉢
- ④ ㉠, ㉢
- ⑤ ㉡, ㉢

해설

- ㉠. $3 + 9 + \cdots + 3^{n-1} = \sum_{k=1}^{n-1} 3^k$ (거짓)
- ㉡. $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + n \cdot 1 = \sum_{k=1}^n k(n-k+1)$ (거짓)
- ㉢. 주어진 수열의 일반항은 $n \cdot 2^{n-1}$ 이므로
 $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 10 \cdot 2^9 = \sum_{k=1}^{10} k \cdot 2^{k-1}$ (참)

19. $\sum_{k=1}^{10} a_k^2 = 20$, $\sum_{k=1}^{10} a_k = 5$ 일 때, $\sum_{k=1}^{10} (2a_k - 3)^2$ 의 값은?

- ① 110 ② 120 ③ 130 ④ 140 ⑤ 150

해설

$$\begin{aligned} & \sum_{k=1}^{10} (2a_k - 3)^2 \\ &= \sum_{k=1}^{10} 4a_k^2 - \sum_{k=1}^{10} 12a_k + \sum_{k=1}^{10} 9 \\ &= 4 \times 20 - 12 \times 5 + 9 \times 10 \\ &= 80 - 60 + 90 = 110 \end{aligned}$$

20. $\sum_{k=1}^{100} a_k = 10$, $\sum_{k=1}^{100} a_k^2 = 20$, 일 때, $\sum_{k=1}^{100} (a_k + 1)^2 + \sum_{k=1}^{100} (a_k - 2)^2$ 의 값은?

- ① 520 ② 540 ③ 560 ④ 580 ⑤ 600

해설

$$\begin{aligned} & \sum_{k=1}^{100} (a_k + 1)^2 + \sum_{k=1}^{100} (a_k - 2)^2 \\ &= \sum_{k=1}^{100} (2a_k^2 - 2a_k + 5) \\ &= 2 \cdot \sum_{k=1}^{100} a_k^2 - 2 \cdot \sum_{k=1}^{100} a_k + \sum_{k=1}^{100} 5 \\ &= 2 \cdot 20 - 2 \cdot 10 + 500 \\ &= 40 - 20 + 500 = 520 \end{aligned}$$