1. $\sum_{k=1}^{10} a_k^2 = 20$, $\sum_{k=1}^{10} a_k = 5$ 일 때, $\sum_{k=1}^{10}(2a_k - 3)^2$ 의 값은?

① 110　② 120　③ 130　④ 140　⑤ 150

해설

$\sum_{k=1}^{10}(2a_k - 3)^2$

$= \sum_{k=1}^{10} 4a_k^2 - \sum_{k=1}^{10} 12a_k + \sum_{k=1}^{10} 9$

$= 4 \times 20 - 12 \times 5 + 9 \times 10$

$= 80 - 60 + 90 = 110$

2. $\sum_{k=1}^{10} a_k = 5$, $\sum_{k=1}^{10} a_k^2 = 20$일 때, $\sum_{k=1}^{10}(a_k+1)^3 - \sum_{k=1}^{10}(a_k-1)^3$의 값은?

① 110 ② 120 ③ 122 ④ 132 ⑤ 140

해설

$\sum_{k=1}^{10}(a_k+1)^3 - \sum_{k=1}^{10}(a_k-1)^3$
$= \sum_{k=1}^{10}(a_k^3+3a_k^2+3a_k+1) - \sum_{k=1}^{10}(a_k^3-3a_k^2+3a_k-1)$
$= \sum_{k=1}^{10}(6a_k^2+2) = 6\sum_{k=1}^{10} a_k^2 + \sum_{k=1}^{10} 2$
$= 6 \times 20 + 2 \times 10 = 140$

3. 두 수열 $\{a_n\}$, $\{b_n\}$에 대하여 $a_n = \frac{n}{3}$, $b_n = 2^n$일 때, $\sum_{k=1}^{5}(a_k + b_k)$의 값은?

① 61 ② 63 ③ 65 ④ 67 ⑤ 69

해설

$\sum_{k=1}^{5}(a_k + b_k) = \sum_{k=1}^{5} a_k + \sum_{k=1}^{5} b_k = \sum_{k=1}^{5} \frac{k}{3} + \sum_{k=1}^{5} 2^k$

$= \frac{1}{3} \cdot \frac{5 \cdot 6}{2} + \frac{2(2^5 - 1)}{2 - 1} = 67$

4. 다음 식의 값은?

$$\sum_{k=1}^{10}(k^2+k)-\sum_{k=4}^{10}(k^2+k)$$

① 14 ② 16 ③ 18 ④ 20 ⑤ 22

해설

(준 식)$=\sum_{k=1}^{3}(k^2+k)=(1^2+1)+(2^2+2)+(3^2+3)=20$

5. $\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 470

해설

$\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2 = \left(\sum_{k=1}^{15} k^2 - \sum_{k=1}^{10} k^2\right) - \sum_{k=1}^{10} k^2$

$= \sum_{k=1}^{15} k^2 - 2\sum_{k=1}^{10} k^2$

$= \dfrac{15 \cdot 16 \cdot 31}{6} - 2 \cdot \dfrac{10 \cdot 11 \cdot 21}{6} = 470$

6. $\sum_{k=1}^{n} a_k = \mathrm{A}$, $\sum_{k=1}^{n} b_k = \mathrm{B}$일 때, 다음 중 옳지 않은 것은?

① $\sum_{k=1}^{n}(a_k + b_k) = \mathrm{A} + \mathrm{B}$

② $\sum_{k=1}^{n}(a_k - b_k) = \mathrm{A} - \mathrm{B}$

③ $\sum_{k=1}^{n} ca_k = c\mathrm{A}$(단, c는 상수)

④ $\sum_{k=2}^{n+1} b_{k-1} = \mathrm{B} - 1$

⑤ $\sum_{k=1}^{n}(a_k + c) = \mathrm{A} + cn$(단, c는 상수)

해설

$\sum_{k=2}^{n+1} b_{k-1} = \sum_{k=1}^{n} b_k = \mathrm{B}$

따라서 , ④가 옳지 않다.

7. 수열 $\{a_n\}$이 $a_1 = 1$, $a_{10} = 30$을 만족할 때 $\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$의 값은?

① 26 ② 27 ③ 28 ④ 29 ⑤ 30

해설

$\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$
$= (a_2 + a_3 + \cdots + a_9 + a_{10}) -$
$(a_1 + a_2 + \cdots + a_9)$
$= -a_1 + a_{10} = -1 + 30 = 29$

8. $\sum_{k=3}^{10} k(k+2)$ 의 값은?

① 460 ② 468 ③ 478 ④ 480 ⑤ 484

해설

$\sum_{k=1}^{10} k(k+2) = \sum_{k=1}^{10} k(k+2) - \sum_{k=1}^{2} k(k+2)$
$= \sum_{k=1}^{10}(k^2+2k) - \sum_{k=1}^{2}(k^2+2k)$
$= \sum_{k=1}^{10} k^2 + 2\sum_{k=1}^{10} k - (3+8)$
$= \frac{10 \cdot 11 \cdot 21}{6} + 2 \cdot \frac{10 \cdot 11}{2} - 11$
$= 385 + 110 - 11$
$= 484$

9. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$ 의 값은?

① 385 ② 550 ③ 1100 ④ 1150 ⑤ 1200

해설

$\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$

$=\sum_{j=1}^{10}\left\{3j+\frac{j(j+1)}{2}\right\}$

$=\sum_{j=1}^{10}\left(\frac{j^2+7i}{2}\right)$

$=\frac{1}{2}\left(\sum_{j=1}^{10}j^2+7\sum_{j=1}^{10}j\right)$

$=\frac{1}{2}\left(\frac{10\cdot 11\cdot 21}{6}+7\times\frac{10\cdot 11}{2}\right)$

$=\frac{1}{2}(385+385)=385$

10. $\sum_{l=1}^{10} \left\{ \sum_{k=1}^{5} (k+l) \right\}$ 의 값은?

① 400　② 425　③ 450　④ 475　⑤ 500

해설

$\sum_{l=1}^{5}(k+l) = \sum_{k=1}^{5} k + \sum_{k=1}^{5} l = \sum_{k=1}^{5} k + 5l$

$\therefore$ (준 식) $= \sum_{l=1}^{10}(5l+15) = 5\sum_{l=1}^{10} l + 150$

$= 5 \times 55 + 150 = 425$

11. $\sum_{k=1}^{n} a_k = 10n$, $\sum_{k=1}^{n} b_k = 5n$ 일 때, $\sum_{n=1}^{10} \left\{ \sum_{k=1}^{n} (2a_k - 3b_k + 5) \right\}$ 의 값은?

① 250　② 300　③ 450　④ 550　⑤ 650

해설

$\sum_{n=1}^{10} \left\{ 2\sum_{k=1}^{n} a_k - 3\sum_{k=1}^{n} b_k + \sum_{k=1}^{n} 5 \right\}$

$= \sum_{n=1}^{10} (2 \cdot 10n - 3 \cdot 5n + 5n)$

$= \sum_{n=1}^{10} (20n - 15n + 5n)$

$= \sum_{n=1}^{10} 10n = 10 \cdot \frac{10 \cdot 11}{2}$

$= 550$

12. $\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$ 의 값은?

① 385 ② 550 ③ 1100 ④ 1150 ⑤ 1200

해설

$\sum_{j=1}^{10}\left\{\sum_{i=1}^{j}(3+i)\right\}$

$=\sum_{j=1}^{10}\left\{3j+\frac{j(j+1)}{2}\right\}$

$=\sum_{j=1}^{10}\left(\frac{j^2+7j}{2}\right)$

$=\frac{1}{2}\left(\sum_{j=1}^{10}j^2+7\cdot\sum_{j=1}^{10}j\right)$

$=\frac{1}{2}\left(\frac{10\cdot 11\cdot 12}{6}+7\times\frac{10\cdot 11}{2}\right)$

$=\frac{1}{2}(385+385)$

$=385$

13. 다음 수열의 합을 $\sum$기호를 써서 나타내면?

$3+6+12+\cdots+3\cdot 2^{n-1}$

① $\sum_{k=1}^{n} 3\cdot 2^{k-1}$　② $\sum_{k=1}^{n-1} 3\cdot 2^{k-1}$　③ $\sum_{k=1}^{n} 3\cdot 2^{k}$
④ $\sum_{k=1}^{n-1} 3\cdot 2^{k}$　⑤ $\sum_{k=1}^{n} 3\cdot 2^{k+1}$

해설

제 k항은 $3\cdot 2^{k-1}$, 항 수는 n이므로
$3+6+9+\cdots+3\cdot 2^{n-1}=\sum_{k=1}^{n} 3\cdot 2^{k-1}$

14. $\sum_{l=1}^{n}\left(\sum_{k=1}^{l}k\right)=364$를 만족하는 n의 값은?

① 10 ② 11 ③ 12 ④ 13 ⑤ 14

해설

$\sum_{l=1}^{n}\left(\sum_{k=1}^{l}k\right)=\sum_{l=1}^{n}\left\{\frac{l(l+1)}{2}\right\}=\frac{1}{2}\sum_{l=1}^{n}(l^2+l)$

$=\frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right\}$

$=\frac{1}{2}\times\frac{n(n+1)(n+2)}{3}$

$=\frac{n(n+1)(n+2)}{6}$

$=364=2^2\times7\times13$

$\therefore\ n(n+1)(n+2)=6\times2^2\times7\times13=12\times13\times14$

따라서 $n=12$

15. 두 수열 a_n, b_n에 대하여 $a_n = n^3 + 3n^2 + 2n$, $b_n = n^2 + n$일 때, $\sum_{i=1}^{4} \left(\sum_{j=1}^{3} a_i b_j\right)$의 값은?

① 4000　② 4100　③ 4200　④ 4300　⑤ 4400

해설

$a_n = n^3 + 3n^2 + 2n = n(n+1)(n+2)$

$b_n = n^2 + n = n(n+1)$

$\therefore \sum_{i=1}^{4} \left(\sum_{j=1}^{3} a_i b_j\right) = \sum_{i=1}^{4} a_i \left(\sum_{j=1}^{3} b_j\right)$

$= \left(\sum_{i=1}^{4} a_i\right) \times \left(\sum_{j=1}^{3} b_j\right)$

$= \left\{\sum_{i=1}^{4} i(i+1)(i+2)\right\} \times \sum_{j=1}^{3} j(j+1)$

$= \sum_{i=1}^{4} (i^3 + 3i^2 + 2i) \times \sum_{j=1}^{3} (j^2 + j)$

$$= \left\{\left(\frac{4\cdot5}{2}\right)^2 + 3\cdot\frac{4\cdot5\cdot9}{6} + 2\cdot\frac{4\cdot5}{2}\right\}$$

$$\times \left(\frac{3\cdot4\cdot7}{6} + \frac{3\cdot4}{2}\right)$$

$= 210 \times 20 = 4200$

16. $\sum_{l=1}^{n}\left(\sum_{k=1}^{l} 12k\right) = 1008$을 만족시키는 n의 값은?

① 5 ② 6 ③ 7 ④ 8 ⑤ 9

해설

$\sum_{l=1}^{n}\left(\sum_{k=1}^{l} 12k\right)$

$= \sum_{l=1}^{n} 12 \cdot \left\{\frac{l(l+1)}{2}\right\} = 6\left(\sum_{l=1}^{n} l^2 + \sum_{l=1}^{n} l\right)$

$= 6\left\{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right\}$

$= n(n+1)(2n+4) = 2n(n+1)(n+2)$

즉, $2n(n+1)(n+2) = 1008$이므로

$n(n+1)(2n+4) = 7 \cdot 8 \cdot 9 = 504$

$\therefore\ n = 7$

17. 다음을 계산하여라.

$$1\cdot 1+2\cdot 4+3\cdot 7+\cdots+10\cdot 28$$

▶ 답 :

▷ 정답 : 1045

해설

$1\cdot 1+2\cdot 4+3\cdot 7+\cdots+10\cdot 28$
$=\sum_{k=1}^{10} k\cdot(3k-2)$
$=\sum_{k=1}^{10}(3k^2-2k)$
$=3\sum_{k=1}^{10} k^2-2\sum_{k=1}^{10} k$
$=3\cdot\dfrac{10\cdot 11\cdot 21}{6}-2\cdot\dfrac{10\cdot 11}{2}$
$=1155-110$
$=1045$

18. $1 \cdot 20 + 2 \cdot 19 + 3 \cdot 18 + \cdots + 20 \cdot 1$ 의 값은?

① 1102 ② 1214 ③ 1368 ④ 1540 ⑤ 1748

해설

$1 \cdot 20 + 2 \cdot 19 + 3 \cdot 18 + \cdots + 20 \cdot 1$
$= \sum_{k=1}^{20} k(21-k) = \sum_{k=1}^{20} (21k - k^2)$
$= 21\sum_{k=1}^{20} k - \sum_{k=1}^{20} k^2$
$= 21 \cdot \frac{20 \cdot 21}{2} - \frac{20 \cdot 21 \cdot 41}{6}$
$= 4410 - 2870 = 1540$

19. 수열 $1 \cdot 2 \cdot 4,\ 2 \cdot 4 \cdot 8,\ 3 \cdot 6 \cdot 12,\ 4 \cdot 8 \cdot 16, \cdots$ 의 제 10항까지의 합은?

① 400 ② 1100 ③ 12100
④ 24200 ⑤ 48400

해설

$a_k = k \cdot 2k \cdot 4k = 8k^3$ 이므로

$S_{10} = \sum_{k=1}^{10} 8k^3 = 8 \cdot \left(\frac{10 \cdot 11}{2}\right)^2 = 2 \cdot 10^2 \cdot 11^2 = 24200$

20. 다음 보기 중 옳은 것을 모두 고른 것은?

보기

㉠ $3+9+\cdots+3^{n-1}=\sum_{k=1}^{n-1}3^{k-1}$

㉡ $1\cdot n+2\cdot(n-1)+3\cdot(n-2)+\cdots+n\cdot 1=\sum_{k=1}^{n}k(n-k)$

㉢ $1\cdot 1+2\cdot 2+3\cdot 2^2+4\cdot 2^3+\cdots+10\cdot 2^9=\sum_{k=1}^{10}k\cdot 2^{k-1}$

① ㉠　② ㉡　③ ㉢　④ ㉠, ㉢　⑤ ㉡, ㉢

해설

㉠. $3+9+\cdots+3^{n-1}=\sum_{k=1}^{n-1}3^{k}$(거짓)
㉡. $1\cdot n+2\cdot(n-1)+3\cdot(n-2)+\cdots+n\cdot 1=\sum_{k=1}^{n}k(n-k+1)$(거짓)
㉢.주어진 수열의 일반항은 $n\cdot 2^{n-1}$ 이므로
$1\cdot 1+2\cdot 2+3\cdot 2^2+4\cdot 2^3+\cdots+10\cdot 2^9=\sum_{k=1}^{10}k\cdot 2^{k-1}$(참)

21. 수열 $1\cdot1,\ 2\cdot3,\ 3\cdot5,\ 4\cdot7, \cdots$ 에서 첫째항부터 제 n항까지의 합은?

① $\frac{1}{6}n(n+1)(n+2)$
② $\frac{1}{6}n(n+1)(2n-2)$
③ $\frac{1}{6}n(n+1)(2n+1)$
④ $\frac{1}{6}n(n+1)(4n-1)$
⑤ $\frac{1}{6}n(n+1)(4n+1)$

해설

주어진 수열의 일반항을 a_k 라 하면

$a_k = k(2k-1) = 2k^2 - k$

$\therefore \sum_{k=1}^{n}(2k^2-k)$

$= 2\cdot\frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1)$

$= \frac{1}{6}n(n+1)\left\{2(2n+1)-3\right\}$

$= \frac{1}{6}n(n+1)(4n-1)$

22. 수열 $1+(1+2)+(1+2+3)+\cdots+(1+2+3+\cdots+n)$의 합을 구하면?

① $\frac{1}{2}n(n+1)(n+2)$ ② $\frac{1}{4}n(n+1)(n+2)$
③ $\frac{1}{6}n(n+1)(n+2)$ ④ $\frac{1}{4}n(n+1)(n+3)$
⑤ $\frac{1}{6}n(n+1)(n+3)$

해설

$a_n = 1+2+\cdots+n$

$= \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$S_n = \sum_{k=1}^{n} \frac{k(k+1)}{2} = \frac{1}{2}\sum_{k=1}^{n}(k^2+k)$

$= \frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right\}$

$= \frac{1}{2}\cdot\frac{n(n+1)(2n+1+3)}{6}$

$= \frac{n(n+1)\cdot 2(n+2)}{2\cdot 6}$

$= \frac{n(n+1)(n+2)}{6}$

23. 이차방정식 $x^2-2x-5=0$의 두 근을 α, β라 할 때, $\sum_{k=1}^{10}(\alpha-k)(\beta-k)$의 값은?

① 215 ② 225 ③ 235 ④ 245 ⑤ 255

해설

이차방정식의 근과 계수의 관계에 의하여

$\alpha+\beta=2,\ \alpha\beta=-5$

$\therefore \sum_{k=1}^{10}(\alpha-k)(\beta-k)$

$=\sum_{k=1}^{10}\{k^2-(\alpha+\beta)k+\alpha\beta\}$

$=\sum_{k=1}^{10}(k^2-2k-5)$

$=\dfrac{10\cdot 11\cdot 21}{6}-2\times\dfrac{10\cdot 11}{2}-50=225$

24. 수열 $\{a_n\}$의 첫째항부터 제n항까지의 합 S_n이 $S_n = n^2 + 2n$일 때, $\sum_{k=1}^{5} ka_k$의 값은?

① 110 ② 125 ③ 145 ④ 160 ⑤ 180

해설

$S_n = n^2 + 2n$이므로

$n \geq 2$일 때,

$a_n = S_n - S_{n-1}$

$= (n^2 + 2n) - \{(n-1)^2 + 2(n-1)\}$

$= 2n + 1 (n = 2,\ 3,\ 4, \cdots)$

$n = 1$ 일 때,

$a_1 = S_1 = 1^2 + 2 \cdot 1 = 3$

따라서

$a_n = 2n + 1 (n = 1,\ 2,\ 3, \cdots)$이므로

$\sum_{k=1}^{5} ka_k = \sum_{k=1}^{5} k(2k+1)$

$= \sum_{k=1}^{5} (2k^2 + k) = 2\sum_{k=1}^{5} k^2 + \sum_{k=1}^{5} k$

$= 2 \cdot \frac{5 \cdot 6 \cdot 11}{6} + \frac{5 \cdot 6}{2} = 125$

25. $\sum_{k=1}^{n} a_k = 2n^2 - n$ 일 때, $\sum_{k=1}^{5}(2k+1)a_k$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 395

해설

$a_n = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n-1} a_k$

$= (2n^2 - n) - \{2(n-1)^2 - (n-1)\}$

$= 4n - 3(n = 2,\ 3,\ 4,\ \cdots)$

$n = 1$ 일 때, $a_1 = 2 \cdot 1^2 - 1 = 1$

따라서 $a_n = 4n - 3(n = 1,\ 2,\ 3,\ \cdots)$ 이므로

$\sum_{k=1}^{5}(2k+2)a_k = \sum_{k=1}^{5}(2k+1)(4k-3)$

$= \sum_{k=1}^{5}(8k^2 - 2k - 3)$

$= 8 \cdot \frac{5 \cdot 6 \cdot 11}{6} - 2 \cdot \frac{5 \cdot 6}{2} - 3 \cdot 5$

$= 440 - 30 - 15 = 395$

26. $\sum_{i=1}^{100} x_i = 4, \sum_{i=1}^{100} y_i = 6$일때, $\sum_{k=1}^{100}(3x_k - 2y_k)$의 값을 구하여라.

▶ 답 :

▷ 정답 : 0

해설

$\sum_{k=1}^{100}(3x_k - 2y_k) = 3\sum_{k=1}^{100} x_k - 2\sum_{k=1}^{100} y_k$
$= 3\sum_{i=1}^{100} x_i - 2\sum_{i=1}^{100} y_i = 3 \cdot 4 - 2 \cdot 6 = 0$

27. $\sum_{k=1}^{10}(11-k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 55

해설

$\sum_{k=1}^{10}(11-k)=10+9+8+\cdots+2+1$

$=\sum_{k=1}^{10}k=\dfrac{10\cdot 11}{2}=55$

28. $a_1 + a_3 + a_5 + \cdots + a_{99}$ 를 $\sum$를 이용하여 나타내면?

① $\sum_{k=1}^{99} a_k$ ② $\sum_{k=1}^{99} a_{2k-1}$ ③ $\sum_{k=1}^{99} a_{2k+1}$

④ $\sum_{k=1}^{50} a_k$ ⑤ $\sum_{k=1}^{50} a_{2k-1}$

해설

① $\sum_{k=1}^{99} a_k = a_1 + a_2 + a_3 + \cdots + a_{99}$

② $\sum_{k=1}^{99} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{197}$

③ $\sum_{k=1}^{99} a_{2k+1} = a_3 + a_5 + a_7 + \cdots + a_{199}$

④ $\sum_{k=1}^{50} a_k = a_1 + a_2 + a_3 + \cdots + a_{50}$

⑤ $\sum_{k=1}^{50} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{99}$

29. 첫째항부터 제 n항까지의 합 S_n이 $S_n = 2n^2 - n + 3$인 수열 $\{a_n\}$에서 $\sum_{k=1}^{5} a_{2k-1}$의 값은?

① 82 ② 84 ③ 86 ④ 88 ⑤ 90

해설

$S_n = 2n^2 - n + 3$이므로

$a_n = S_n - S_{n-1}$

$= 2n^2 - n + 3 - \{2(n-1)^2 - (n-1) + 3\}$

$= 4n - 3$ $(n \geq 2)$

$a_1 = S_1 = 2 - 1 + 3 = 4$

$\therefore \sum_{k=1}^{5} a_{2k-1} = a_1 + a_3 + a_5 + a_7 + a_9$

$= 4 + 9 + 17 + 25 + 33 = 88$

30. 수열 $\{a_n\}$에 대하여 $\sum_{k=1}^{n} a_k = n^2 + n$일 때, $\sum_{k=1}^{n} a_{2k-1}$을 n에 대한 식으로 나타내면?

① $n^2 + 1$ ② $n^2 + 3n$ ③ $2n^2$
④ $2n^2 + n$ ⑤ $3n^2 - 1$

해설

$\sum_{k=1}^{n} a_k = n^2 + n$이므로
$n \geq 2$일 때,

$a_n = S_n - S_{n-1}$
$= n^2 + n - \{(n-1)^2 + (n-1)\}$
$= 2n \cdots\cdots$ ㉠

$n = 1$일 때, $a_1 = S_1 = 2$

이것은 ㉠에 $n = 1$을 대입하여 얻은 값과 같으므로 수열 $\{a_n\}$의 일반항은
$a_n = 2n$
$\therefore a_{2k-1} = 2(2k-1) = 4k - 2$
$\therefore \sum_{k=1}^{n} a_{2k-1} = \sum_{k=1}^{n} (4k-2)$
$= 4 \cdot \frac{n(n+1)}{2} - 2n$
$= 2n^2$