

1. $\sum_{k=1}^{10} a_k^2 = 20$, $\sum_{k=1}^{10} a_k = 5$ 일 때, $\sum_{k=1}^{10} (2a_k - 3)^2$ 의 값은?

① 110

② 120

③ 130

④ 140

⑤ 150

해설

$$\begin{aligned} & \sum_{k=1}^{10} (2a_k - 3)^2 \\ &= \sum_{k=1}^{10} 4a_k^2 - \sum_{k=1}^{10} 12a_k + \sum_{k=1}^{10} 9 \\ &= 4 \times 20 - 12 \times 5 + 9 \times 10 \\ &= 80 - 60 + 90 = 110 \end{aligned}$$

2. $\sum_{k=1}^{10} a_k = 5$, $\sum_{k=1}^{10} a_k^2 = 20$ 일 때, $\sum_{k=1}^{10} (a_k + 1)^3 - \sum_{k=1}^{10} (a_k - 1)^3$ 의 값은?

① 110

② 120

③ 122

④ 132

⑤ 140

해설

$$\begin{aligned} & \sum_{k=1}^{10} (a_k + 1)^3 - \sum_{k=1}^{10} (a_k - 1)^3 \\ &= \sum_{k=1}^{10} (a_k^3 + 3a_k^2 + 3a_k + 1) - \sum_{k=1}^{10} (a_k^3 - 3a_k^2 + 3a_k - 1) \\ &= \sum_{k=1}^{10} (6a_k^2 + 2) = 6 \sum_{k=1}^{10} a_k^2 + \sum_{k=1}^{10} 2 \\ &= 6 \times 20 + 2 \times 10 = 140 \end{aligned}$$

3. 두 수열 $\{a_n\}, \{b_n\}$ 에 대하여 $a_n = \frac{n}{3}$, $b_n = 2^n$ 일 때, $\sum_{k=1}^5 (a_k + b_k)$ 의 값은?

① 61

② 63

③ 65

④ 67

⑤ 69

해설

$$\begin{aligned}\sum_{k=1}^5 (a_k + b_k) &= \sum_{k=1}^5 a_k + \sum_{k=1}^5 b_k = \sum_{k=1}^5 \frac{k}{3} + \sum_{k=1}^5 2^k \\ &= \frac{1}{3} \cdot \frac{5 \cdot 6}{2} + \frac{2(2^5 - 1)}{2 - 1} = 67\end{aligned}$$

4. 다음 식의 값은?

$$\sum_{k=1}^{10}(k^2 + k) - \sum_{k=4}^{10}(k^2 + k)$$

① 14

② 16

③ 18

④ 20

⑤ 22

해설

$$(\text{준 식}) = \sum_{k=1}^3 (k^2 + k) = (1^2 + 1) + (2^2 + 2) + (3^2 + 3) = 20$$

5. $\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 470

해설

$$\begin{aligned}\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2 &= (\sum_{k=1}^{15} k^2 - \sum_{k=1}^{10} k^2) - \sum_{k=1}^{10} k^2 \\ &= \sum_{k=1}^{15} k^2 - 2 \sum_{k=1}^{10} k^2 \\ &= \frac{15 \cdot 16 \cdot 31}{6} - 2 \cdot \frac{10 \cdot 11 \cdot 21}{6} = 470\end{aligned}$$

6. $\sum_{k=1}^n a_k = A$, $\sum_{k=1}^n b_k = B$ 일 때, 다음 중 옳지 않은 것은?

① $\sum_{k=1}^n (a_k + b_k) = A + B$

② $\sum_{k=1}^n (a_k - b_k) = A - B$

③ $\sum_{k=1}^n ca_k = cA$ (단, c 는 상수)

④ $\sum_{k=2}^{n+1} b_{k-1} = B - 1$

⑤ $\sum_{k=1}^n (a_k + c) = A + cn$ (단, c 는 상수)

해설

$$\sum_{k=2}^{n+1} b_{k-1} = \sum_{k=1}^n b_k = B$$

따라서, ④가 옳지 않다.

7. 수열 $\{a_n\}$ 이 $a_1 = 1$, $a_{10} = 30$ 을 만족할 때 $\sum_{k=1}^9 a_{k+1} - \sum_{k=2}^{10} a_{k-1}$ 의 값은?

① 26

② 27

③ 28

④ 29

⑤ 30

해설

$$\begin{aligned} & \sum_{k=1}^9 a_{k+1} - \sum_{k=2}^{10} a_{k-1} \\ &= (a_2 + a_3 + \cdots + a_9 + a_{10}) - \\ & \quad (a_1 + a_2 + \cdots + a_9) \\ &= -a_1 + a_{10} = -1 + 30 = 29 \end{aligned}$$

8. $\sum_{k=3}^{10} k(k+2)$ 의 값은?

① 460

② 468

③ 478

④ 480

⑤ 484

해설

$$\begin{aligned}\sum_{k=1}^{10} k(k+2) &= \sum_{k=1}^{10} k(k+2) - \sum_{k=1}^2 k(k+2) \\&= \sum_{k=1}^{10} (k^2 + 2k) - \sum_{k=1}^2 (k^2 + 2k) \\&= \sum_{k=1}^{10} k^2 + 2 \sum_{k=1}^{10} k - (3 + 8) \\&= \frac{10 \cdot 11 \cdot 21}{6} + 2 \cdot \frac{10 \cdot 11}{2} - 11 \\&= 385 + 110 - 11 \\&= 484\end{aligned}$$

9. $\sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\}$ 의 값은?

① 385

② 550

③ 1100

④ 1150

⑤ 1200

해설

$$\begin{aligned} & \sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\} \\ &= \sum_{j=1}^{10} \left\{ 3j + \frac{j(j+1)}{2} \right\} \\ &= \sum_{j=1}^{10} \left(\frac{j^2 + 7j}{2} \right) \\ &= \frac{1}{2} \left(\sum_{j=1}^{10} j^2 + 7 \sum_{j=1}^{10} j \right) \\ &= \frac{1}{2} \left(\frac{10 \cdot 11 \cdot 21}{6} + 7 \times \frac{10 \cdot 11}{2} \right) \\ &= \frac{1}{2} (385 + 385) = 385 \end{aligned}$$

10. $\sum_{l=1}^{10} \{ \sum_{k=1}^5 (k+l) \}$ 의 값은?

① 400

② 425

③ 450

④ 475

⑤ 500

해설

$$\sum_{l=1}^5 (k+l) = \sum_{k=1}^5 k + \sum_{k=1}^5 l = \sum_{k=1}^5 k + 5l$$

$$\begin{aligned} \therefore (\text{준 식}) &= \sum_{l=1}^{10} (5l + 15) = 5 \sum_{l=1}^{10} l + 150 \\ &= 5 \times 55 + 150 = 425 \end{aligned}$$

11. $\sum_{k=1}^n a_k = 10n$, $\sum_{k=1}^n b_k = 5n$ 일 때, $\sum_{n=1}^{10} \left\{ \sum_{k=1}^n (2a_k - 3b_k + 5) \right\}$ 의 값은?

① 250

② 300

③ 450

④ 550

⑤ 650

해설

$$\begin{aligned} & \sum_{n=1}^{10} \left\{ 2 \sum_{k=1}^n a_k - 3 \sum_{k=1}^n b_k + \sum_{k=1}^n 5 \right\} \\ &= \sum_{n=1}^{10} (2 \cdot 10n - 3 \cdot 5n + 5n) \\ &= \sum_{n=1}^{10} (20n - 15n + 5n) \\ &= \sum_{n=1}^{10} 10n = 10 \cdot \frac{10 \cdot 11}{2} \\ &= 550 \end{aligned}$$

12. $\sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\}$ 의 값은?

① 385

② 550

③ 1100

④ 1150

⑤ 1200

해설

$$\begin{aligned} & \sum_{j=1}^{10} \left\{ \sum_{i=1}^j (3+i) \right\} \\ &= \sum_{j=1}^{10} \left\{ 3j + \frac{j(j+1)}{2} \right\} \\ &= \sum_{j=1}^{10} \left(\frac{j^2 + 7j}{2} \right) \\ &= \frac{1}{2} \left(\sum_{j=1}^{10} j^2 + 7 \cdot \sum_{j=1}^{10} j \right) \\ &= \frac{1}{2} \left(\frac{10 \cdot 11 \cdot 12}{6} + 7 \times \frac{10 \cdot 11}{2} \right) \\ &= \frac{1}{2} (385 + 385) \\ &= 385 \end{aligned}$$

13. 다음 수열의 합을 $\sum$ 기호를 써서 나타내면?

$$3 + 6 + 12 + \cdots + 3 \cdot 2^{n-1}$$

① $\sum_{k=1}^n 3 \cdot 2^{k-1}$

② $\sum_{k=1}^{n-1} 3 \cdot 2^{k-1}$

③ $\sum_{k=1}^n 3 \cdot 2^k$

④ $\sum_{k=1}^{n-1} 3 \cdot 2^k$

⑤ $\sum_{k=1}^n 3 \cdot 2^{k+1}$

해설

제 k 항은 $3 \cdot 2^{k-1}$, 항 수는 n 이므로

$$3 + 6 + 9 + \cdots + 3 \cdot 2^{n-1} = \sum_{k=1}^n 3 \cdot 2^{k-1}$$

14. $\sum_{l=1}^n (\sum_{k=1}^l k) = 364$ 를 만족하는 n 의 값은?

① 10

② 11

③ 12

④ 13

⑤ 14

해설

$$\sum_{l=1}^n (\sum_{k=1}^l k) = \sum_{l=1}^n \left\{ \frac{l(l+1)}{2} \right\} = \frac{1}{2} \sum_{l=1}^n (l^2 + l)$$

$$= \frac{1}{2} \left\{ \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right\}$$

$$= \frac{1}{2} \times \frac{n(n+1)(n+2)}{3}$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$= 364 = 2^2 \times 7 \times 13$$

$$\therefore n(n+1)(n+2) = 6 \times 2^2 \times 7 \times 13 = 12 \times 13 \times 14$$

따라서 $n = 12$

15. 두 수열 a_n, b_n 에 대하여 $a_n = n^3 + 3n^2 + 2n$, $b_n = n^2 + n$ 일 때,
 $\sum_{i=1}^4 (\sum_{j=1}^3 a_i b_j)$ 의 값은?

① 4000

② 4100

③ 4200

④ 4300

⑤ 4400

해설

$$a_n = n^3 + 3n^2 + 2n = n(n+1)(n+2)$$

$$b_n = n^2 + n = n(n+1)$$

$$\therefore \sum_{i=1}^4 (\sum_{j=1}^3 a_i b_j) = \sum_{i=1}^4 a_i (\sum_{j=1}^3 b_j)$$

$$= (\sum_{i=1}^4 a_i) \times (\sum_{j=1}^3 b_j)$$

$$= \left\{ \sum_{i=1}^4 i(i+1)(i+2) \right\} \times \sum_{j=1}^3 j(j+1)$$

$$= \sum_{i=1}^4 (i^3 + 3i^2 + 2i) \times \sum_{j=1}^3 (j^2 + j)$$

$$= \left\{ \left(\frac{4 \cdot 5}{2} \right)^2 + 3 \cdot \frac{4 \cdot 5 \cdot 9}{6} + 2 \cdot \frac{4 \cdot 5}{2} \right\}$$

$$\times \left(\frac{3 \cdot 4 \cdot 7}{6} + \frac{3 \cdot 4}{2} \right)$$

$$= 210 \times 20 = 4200$$

16. $\sum_{l=1}^n (\sum_{k=1}^l 12k) = 1008$ 을 만족시키는 n 의 값은?

① 5

② 6

③ 7

④ 8

⑤ 9

해설

$$\begin{aligned} & \sum_{l=1}^n (\sum_{k=1}^l 12k) \\ &= \sum_{l=1}^n 12 \cdot \left\{ \frac{l(l+1)}{2} \right\} = 6 \left(\sum_{l=1}^n l^2 + \sum_{l=1}^n l \right) \\ &= 6 \left\{ \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right\} \\ &= n(n+1)(2n+4) = 2n(n+1)(n+2) \\ &\text{즉, } 2n(n+1)(n+2) = 1008 \text{ 이므로} \\ &n(n+1)(2n+4) = 7 \cdot 8 \cdot 9 = 504 \\ &\therefore n = 7 \end{aligned}$$

17. 다음을 계산하여라.

$$1 \cdot 1 + 2 \cdot 4 + 3 \cdot 7 + \cdots + 10 \cdot 28$$

▶ 답 :

▷ 정답 : 1045

해설

$$1 \cdot 1 + 2 \cdot 4 + 3 \cdot 7 + \cdots + 10 \cdot 28$$

$$= \sum_{k=1}^{10} k \cdot (3k - 2)$$

$$= \sum_{k=1}^{10} (3k^2 - 2k)$$

$$= 3 \sum_{k=1}^{10} k^2 - 2 \sum_{k=1}^{10} k$$

$$= 3 \cdot \frac{10 \cdot 11 \cdot 21}{6} - 2 \cdot \frac{10 \cdot 11}{2}$$

$$= 1155 - 110$$

$$= 1045$$

18. $1 \cdot 20 + 2 \cdot 19 + 3 \cdot 18 + \cdots + 20 \cdot 1$ 의 값은?

① 1102

② 1214

③ 1368

④ 1540

⑤ 1748

해설

$$\begin{aligned} & 1 \cdot 20 + 2 \cdot 19 + 3 \cdot 18 + \cdots + 20 \cdot 1 \\ &= \sum_{k=1}^{20} k(21-k) = \sum_{k=1}^{20} (21k - k^2) \\ &= 21 \sum_{k=1}^{20} k - \sum_{k=1}^{20} k^2 \\ &= 21 \cdot \frac{20 \cdot 21}{2} - \frac{20 \cdot 21 \cdot 41}{6} \\ &= 4410 - 2870 = 1540 \end{aligned}$$

19. 수열 $1 \cdot 2 \cdot 4, 2 \cdot 4 \cdot 8, 3 \cdot 6 \cdot 12, 4 \cdot 8 \cdot 16, \dots$ 의 제 10항까지의 합은?

① 400

② 1100

③ 12100

④ 24200

⑤ 48400

해설

$a_k = k \cdot 2k \cdot 4k = 8k^3$ 이므로

$$S_{10} = \sum_{k=1}^{10} 8k^3 = 8 \cdot \left(\frac{10 \cdot 11}{2} \right)^2 = 2 \cdot 10^2 \cdot 11^2 = 24200$$

20. 다음 보기 중 옳은 것을 모두 고른 것은?

보기

㉠ $3 + 9 + \cdots + 3^{n-1} = \sum_{k=1}^{n-1} 3^{k-1}$

㉡ $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + n \cdot 1 = \sum_{k=1}^n k(n-k)$

㉢ $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 10 \cdot 2^9 = \sum_{k=1}^{10} k \cdot 2^{k-1}$

① ㉠

② ㉡

③ ㉢

④ ㉠, ㉢

⑤ ㉡, ㉢

해설

㉠. $3 + 9 + \cdots + 3^{n-1} = \sum_{k=1}^{n-1} 3^k$ (거짓)

㉡. $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + n \cdot 1 = \sum_{k=1}^n k(n-k+1)$ (거짓)

㉢. 주어진 수열의 일반항은 $n \cdot 2^{n-1}$ 이므로

$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 10 \cdot 2^9 = \sum_{k=1}^{10} k \cdot 2^{k-1}$ (참)

21. 수열 $1 \cdot 1, 2 \cdot 3, 3 \cdot 5, 4 \cdot 7, \dots$ 에서 첫째항부터 제 n 항까지의 합은?

① $\frac{1}{6}n(n+1)(n+2)$

② $\frac{1}{6}n(n+1)(2n-2)$

③ $\frac{1}{6}n(n+1)(2n+1)$

④ $\frac{1}{6}n(n+1)(4n-1)$

⑤ $\frac{1}{6}n(n+1)(4n+1)$

해설

주어진 수열의 일반항을 a_k 라 하면

$$a_k = k(2k-1) = 2k^2 - k$$

$$\therefore \sum_{k=1}^n (2k^2 - k)$$

$$= 2 \cdot \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1)$$

$$= \frac{1}{6}n(n+1) \{2(2n+1) - 3\}$$

$$= \frac{1}{6}n(n+1)(4n-1)$$

22. 수열 $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + 3 + \cdots + n)$ 의 합을 구하면?

① $\frac{1}{2}n(n+1)(n+2)$

② $\frac{1}{4}n(n+1)(n+2)$

③ $\frac{1}{6}n(n+1)(n+2)$

④ $\frac{1}{4}n(n+1)(n+3)$

⑤ $\frac{1}{6}n(n+1)(n+3)$

해설

$$\begin{aligned}
 a_n &= 1 + 2 + \cdots + n \\
 &= \sum_{k=1}^n k = \frac{n(n+1)}{2} \\
 S_n &= \sum_{k=1}^n \frac{k(k+1)}{2} = \frac{1}{2} \sum_{k=1}^n (k^2 + k) \\
 &= \frac{1}{2} \left\{ \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right\} \\
 &= \frac{1}{2} \cdot \frac{n(n+1)(2n+1+3)}{6} \\
 &= \frac{n(n+1) \cdot 2(n+2)}{2 \cdot 6} \\
 &= \frac{n(n+1)(n+2)}{6}
 \end{aligned}$$

23. 이차방정식 $x^2 - 2x - 5 = 0$ 의 두 근을 α, β 라 할 때, $\sum_{k=1}^{10} (\alpha - k)(\beta - k)$ 의 값은?

① 215

② 225

③ 235

④ 245

⑤ 255

해설

이차방정식의 근과 계수의 관계에 의하여

$$\alpha + \beta = 2, \alpha\beta = -5$$

$$\therefore \sum_{k=1}^{10} (\alpha - k)(\beta - k)$$

$$= \sum_{k=1}^{10} \{k^2 - (\alpha + \beta)k + \alpha\beta\}$$

$$= \sum_{k=1}^{10} (k^2 - 2k - 5)$$

$$= \frac{10 \cdot 11 \cdot 21}{6} - 2 \times \frac{10 \cdot 11}{2} - 50 = 225$$

24. 수열 $\{a_n\}$ 의 첫째항부터 제 n 항까지의 합 S_n 이 $S_n = n^2 + 2n$ 일 때, $\sum_{k=1}^5 ka_k$ 의 값은?

① 110

② 125

③ 145

④ 160

⑤ 180

해설

$$S_n = n^2 + 2n \text{ 이므로}$$

$n \geq 2$ 일 때,

$$a_n = S_n - S_{n-1}$$

$$= (n^2 + 2n) - \{(n-1)^2 + 2(n-1)\}$$

$$= 2n + 1 (n = 2, 3, 4, \dots)$$

$n = 1$ 일 때,

$$a_1 = S_1 = 1^2 + 2 \cdot 1 = 3$$

따라서

$$a_n = 2n + 1 (n = 1, 2, 3, \dots) \text{ 이므로}$$

$$\sum_{k=1}^5 ka_k = \sum_{k=1}^5 k(2k+1)$$

$$= \sum_{k=1}^5 (2k^2 + k) = 2 \sum_{k=1}^5 k^2 + \sum_{k=1}^5 k$$

$$= 2 \cdot \frac{5 \cdot 6 \cdot 11}{6} + \frac{5 \cdot 6}{2} = 125$$

25. $\sum_{k=1}^n a_k = 2n^2 - n$ 일 때, $\sum_{k=1}^5 (2k+1)a_k$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 395

해설

$$\begin{aligned} a_n &= \sum_{k=1}^n a_k - \sum_{k=1}^{n-1} a_k \\ &= (2n^2 - n) - \{2(n-1)^2 - (n-1)\} \\ &= 4n - 3 \quad (n = 2, 3, 4, \dots) \end{aligned}$$

$$n = 1 \text{ 일 때, } a_1 = 2 \cdot 1^2 - 1 = 1$$

따라서 $a_n = 4n - 3 \quad (n = 1, 2, 3, \dots)$ 이므로

$$\begin{aligned} \sum_{k=1}^5 (2k+1)a_k &= \sum_{k=1}^5 (2k+1)(4k-3) \\ &= \sum_{k=1}^5 (8k^2 - 2k - 3) \\ &= 8 \cdot \frac{5 \cdot 6 \cdot 11}{6} - 2 \cdot \frac{5 \cdot 6}{2} - 3 \cdot 5 \\ &= 440 - 30 - 15 = 395 \end{aligned}$$

26. $\sum_{i=1}^{100} x_i = 4$, $\sum_{i=1}^{100} y_i = 6$ 일때, $\sum_{k=1}^{100} (3x_k - 2y_k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 0

해설

$$\begin{aligned}\sum_{k=1}^{100} (3x_k - 2y_k) &= 3 \sum_{k=1}^{100} x_k - 2 \sum_{k=1}^{100} y_k \\ &= 3 \sum_{i=1}^{100} x_i - 2 \sum_{i=1}^{100} y_i = 3 \cdot 4 - 2 \cdot 6 = 0\end{aligned}$$

27. $\sum_{k=1}^{10} (11 - k)$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 55

해설

$$\begin{aligned}\sum_{k=1}^{10} (11 - k) &= 10 + 9 + 8 + \cdots + 2 + 1 \\ &= \sum_{k=1}^{10} k = \frac{10 \cdot 11}{2} = 55\end{aligned}$$

28. $a_1 + a_3 + a_5 + \cdots + a_{99}$ 를 $\sum$ 를 이용하여 나타내면?

① $\sum_{k=1}^{99} a_k$

② $\sum_{k=1}^{99} a_{2k-1}$

③ $\sum_{k=1}^{99} a_{2k+1}$

④ $\sum_{k=1}^{50} a_k$

⑤ $\sum_{k=1}^{50} a_{2k-1}$

해설

① $\sum_{k=1}^{99} a_k = a_1 + a_2 + a_3 + \cdots + a_{99}$

② $\sum_{k=1}^{99} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{197}$

③ $\sum_{k=1}^{99} a_{2k+1} = a_3 + a_5 + a_7 + \cdots + a_{199}$

④ $\sum_{k=1}^{50} a_k = a_1 + a_2 + a_3 + \cdots + a_{50}$

⑤ $\sum_{k=1}^{50} a_{2k-1} = a_1 + a_3 + a_5 + \cdots + a_{99}$

29. 첫째항부터 제 n 항까지의 합 S_n 이 $S_n = 2n^2 - n + 3$ 인 수열 $\{a_n\}$ 에서 $\sum_{k=1}^5 a_{2k-1}$ 의 값은?

① 82

② 84

③ 86

④ 88

⑤ 90

해설

$S_n = 2n^2 - n + 3$ 이므로

$$a_n = S_n - S_{n-1}$$

$$= 2n^2 - n + 3 - \{2(n-1)^2 - (n-1) + 3\}$$

$$= 4n - 3 \quad (n \geq 2)$$

$$a_1 = S_1 = 2 - 1 + 3 = 4$$

$$\therefore \sum_{k=1}^5 a_{2k-1} = a_1 + a_3 + a_5 + a_7 + a_9$$

$$= 4 + 9 + 17 + 25 + 33 = 88$$

30. 수열 $\{a_n\}$ 에 대하여 $\sum_{k=1}^n a_k = n^2 + n$ 일 때, $\sum_{k=1}^n a_{2k-1}$ 을 n 에 대한 식으로 나타내면?

① $n^2 + 1$

② $n^2 + 3n$

③ $2n^2$

④ $2n^2 + n$

⑤ $3n^2 - 1$

해설

$$\sum_{k=1}^n a_k = n^2 + n \text{ 이므로}$$

$n \geq 2$ 일 때,

$$\begin{aligned} a_n &= S_n - S_{n-1} \\ &= n^2 + n - \{(n-1)^2 + (n-1)\} \\ &= 2n \dots\dots \textcircled{7} \end{aligned}$$

$$n = 1 \text{ 일 때, } a_1 = S_1 = 2$$

이것은 ㉞에 $n = 1$ 을 대입하여 얻은 값과 같으므로 수열 $\{a_n\}$ 의 일반항은

$$a_n = 2n$$

$$\therefore a_{2k-1} = 2(2k-1) = 4k-2$$

$$\begin{aligned} \therefore \sum_{k=1}^n a_{2k-1} &= \sum_{k=1}^n (4k-2) \\ &= 4 \cdot \frac{n(n+1)}{2} - 2n \\ &= 2n^2 \end{aligned}$$