1. $\log_2(x-4)^2$ 의 값이 존재하기 위한 x의 범위는?

① $x<1$ ② $x>3$ ③ $x<4$ ④ $x \neq 4$ ⑤ $x \neq 5$

해설

$(x-4)^2>0$ 로부터 $x \neq 4$

2. $(\log_3 2)(\log_4 25) - \log_9 75$ 의 값은?

① $-\frac{1}{2}$ ② -1 ③ 0 ④ $\log_3 2$ ⑤ $\log_2 3$

해설

$(\log_3 2)(\log_4 25) - \log_9 75$
$= (\log_3 2)(\log_2 5) - \log_9 75$
$= \log_3 5 - \frac{1}{2}\log_3 75$
$= \log_3 \frac{5}{5\sqrt{3}}$
$= \log_3 \frac{1}{\sqrt{3}}$
$= -\frac{1}{2}$

3. $2\log_3 \frac{2}{3} + \log_3 \sqrt{72} - \frac{1}{2}\log_3 8$을 간단히 하면?

① $\log_3 2$ ② $\log_3 2 - 1$ ③ $2\log_3 2 - 1$

④ $\log_3 +1$ ⑤ $2\log_3 2$

해설

$2\log_3 \frac{2}{3} + \log_3 \sqrt{72} - \frac{1}{2}\log_3 8$

$= \log_3 \left(\frac{2}{3}\right)^2 + \log_3 6\sqrt{2} - \log_3 \sqrt{8}$

$= \log_3 \frac{4}{9} + \log_3 6\sqrt{2} - \log_3 2\sqrt{2}$

$= \log_3 \left(\frac{4}{9} \times 6\sqrt{2} \times \frac{1}{2\sqrt{2}}\right)$

$= \log_3 \frac{4}{3}$

$= 2\log_3 2 - 1$

4. $\log_2(\log_8 x) = -1$ 을 만족하는 x 의 값을 구하여라.

▶ 답 :

▷ 정답 : $2\sqrt{2}$

해설

$\log_2(\log_8 x) = -1$ 에서

$\log_8 x = 2^{-1} = \frac{1}{2}$

$\therefore x = 8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} = 2^{\frac{3}{2}} = 2\sqrt{2}$

5. 다음 중 계산 결과가 다른 하나는?

① $9^{\log_9 4}$

② $\log_{\sqrt{5}} 25$

③ $\log_{\frac{1}{2}} \frac{1}{16}$

④ $\log_{\frac{1}{3}} 81$

⑤ $\log_2 3 \cdot \log_3 5 \cdot \log_5 16$

해설

① $9^{\log_9 4} = 4$

② $\log_{\sqrt{5}} 25 = \log_{5^{\frac{1}{2}}} 5^2 = \frac{2}{1} \log_5 5 = 4$

③ $\log_{\frac{1}{2}} \frac{1}{16} = \log_{2^{-1}} 2^{-4} = \frac{-4}{-1} \log_2 2 = 4$

④ $\log_{\frac{1}{3}} 81 = \log_{3^{-1}} 3^4 = \frac{4}{-1} \log_3 3 = -4$

⑤ $\log_2 3 \cdot \log_3 5 \cdot \log_5 16$

$= \frac{\log_{10} 3}{\log_{10} 2} \cdot \frac{\log_{10} 5}{\log_{10} 3} \cdot \frac{\log_{10} 16}{\log_{10} 5}$

$= \frac{\log_{10} 16}{\log_{10} 2} = \log_2 16 = \log_2 2^4$

$= 4 \log_2 2 = 4$

6. $\frac{1}{2}\log_3 \frac{9}{7} + \log_3 \sqrt{7} = a$, $\log_3 4 \cdot \log_4 \sqrt{3} = b$일 때, $a + 2b$의 값을 구하여라.

▶ 답 :

▷ 정답 : 2

해설

$a = \log_3 \frac{3}{\sqrt{7}} + \log_3 \sqrt{7} = \log_3 3 = 1$

$b = \log_3 4 \cdot \log_4 3^{\frac{1}{2}} = \frac{1}{2}$

$\therefore a + 2b = 1 + 1 = 2$

7. 방정식 $2x^2-8x-1=0$의 두 근이 $\log_{10} a$, $\log_{10} b$일 때, $\log_a b+\log_b a$의 값은?

① -2 ② -8 ③ -12 ④ -26 ⑤ 34

해설

이차방정식의 근과 계수와의 관계에 의하여

$\log_{10} a + \log_{10} b = 4$,

$\log_{10} a \cdot \log_{10} b = -\frac{1}{2}$

$\therefore \log_a b + \log_b a = \frac{\log_{10} b}{\log_{10} a} + \frac{\log_{10} a}{\log_{10} b}$

$= \frac{(\log_{10} a + \log_{10} b)^2 - 2\log_{10} a \cdot \log_{10} b}{\log_{10} a \cdot \log_{10} b}$

$= \frac{16+1}{-\frac{1}{2}} = -34$

8. $\log_2 5$ 의 정수부분을 x, 소수부분을 y 라 할 때, $\dfrac{2^{-x}+2^{-y}}{2^x+2^y}$ 의 값은?

① $\dfrac{1}{5}$ ② $\dfrac{1}{4}$ ③ $\dfrac{1}{3}$ ④ $\dfrac{1}{2}$ ⑤ 2

해설

$\log_2 4 < \log_2 5 < \log_2 8$ 이므로

$x = 2,\ y = \log_2 5 - 2 = \log_2 \frac{5}{4}$

$2^{-x} = 2^{-2} = \dfrac{1}{4},\ 2^x = 4$

$2^{-y} = 2^{-\log_2 \frac{5}{4}} = 2^{\log_2 (\frac{5}{4})^{-1}} = 2^{\log_2 \frac{4}{5}} = \dfrac{4}{5}$

$2^y = 2^{\log_2 \frac{5}{4}} = \dfrac{5}{4}$

$\therefore\ \dfrac{2^{-x}+2^{-y}}{2^x+2^y} = \dfrac{\frac{1}{4}+\frac{4}{5}}{4+\frac{5}{4}} = \dfrac{\frac{21}{20}}{\frac{21}{4}} = \dfrac{4}{20} = \dfrac{1}{5}$