1. $1+\dfrac{1}{1+2}+\dfrac{1}{1+2+3}+\cdots+\dfrac{1}{1+2+3+\cdots+10}$ 의 값은?

① $\dfrac{9}{10}$　② $\dfrac{11}{10}$　③ $\dfrac{10}{11}$　④ $\dfrac{20}{11}$　⑤ $\dfrac{11}{20}$

해설

$$\frac{1}{1+2+\cdots+n}=\frac{1}{\frac{n(n+1)}{2}}=\frac{2}{n(n+1)}$$

$$\therefore \sum_{k=1}^{10}\frac{2}{k(k+1)}=2\sum_{k=1}^{10}\left(\frac{1}{k}-\frac{1}{k+1}\right)$$

$$=2\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{10}-\frac{1}{11}\right)\right\}$$

$$=2\left(1-\frac{1}{11}\right)=\frac{20}{11}$$

2. 수열 $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \cdots$ 의 첫째항부터 제 50까지의 합은?

① $\frac{48}{49}$ ② $\frac{50}{49}$ ③ $\frac{49}{50}$ ④ $\frac{51}{50}$ ⑤ $\frac{50}{51}$

해설

$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \cdots = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$

따라서, 이 수열의 첫째항부터 제 50항까지의 합은

$\sum_{k=1}^{50} \frac{1}{k(k+1)}$

$= \sum_{k=1}^{50} \left(\frac{1}{k} - \frac{1}{k+1}\right)$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{50} - \frac{1}{51}\right)$

$= 1 - \frac{1}{51} = \frac{50}{51}$

3. 다음 식의 값은?

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

① 9 ② $3\sqrt{11}-\sqrt{2}$ ③ $\sqrt{99}-1$

④ $\sqrt{101}-1$ ⑤ 11

해설

$$(\text{준식})=\sum_{k=1}^{99}\frac{1}{\sqrt{k}+\sqrt{k-1}}=\sum_{k=1}^{99}(\sqrt{k+1}-\sqrt{k})$$

$$=(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+\cdots+(\sqrt{100}-\sqrt{99})$$

$$=\sqrt{100}-1=9$$

4. $\sum_{k=1}^{n} \dfrac{1}{\sqrt{k}+\sqrt{k+1}}$ 의 값은?

① $\sqrt{n-1}-1$　　② $\sqrt{n+1}-1$　　③ $\sqrt{n+1}$

④ $\sqrt{n+1}+1$　　⑤ $\sqrt{2n+1}+1$

해설

$$\frac{1}{\sqrt{k}+\sqrt{k+1}} = \frac{\sqrt{k+1}-\sqrt{k}}{(\sqrt{k+1}+\sqrt{k})(\sqrt{k+1}-\sqrt{k})}$$

$$= \frac{\sqrt{k+1}-\sqrt{k}}{(k+1)-k} = \sqrt{k+1}-\sqrt{k}$$

따라서

(주어진 식)$=\sum_{k=1}^{n}(\sqrt{k+1}-\sqrt{k})$

$$= (\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{n+1}-\sqrt{n})$$

$$= \sqrt{n+1}-1$$

5. $1+\dfrac{1}{1+2}+\dfrac{1}{1+2+3}+\cdots+\dfrac{1}{1+2+\cdots+2015}$ 의 값은?

① $\dfrac{2014}{2015}$ ② $\dfrac{2015}{2016}$ ③ $\dfrac{2015}{1008}$ ④ $\dfrac{2014}{1008}$ ⑤ 2

해설

$\dfrac{1}{1+2+\cdots+n}=\dfrac{1}{\dfrac{n(n+1)}{2}}=\dfrac{2}{n(n+1)}$ 이므로

(주어진 식)$=\sum_{k=1}^{2015}\dfrac{2}{n(n+1)}$

$=\sum_{k=1}^{2015}2\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right)$

$=2\left(1-\dfrac{1}{2016}\right)=\dfrac{2\times2015}{2016}=\dfrac{2015}{1008}$

6. 함수 $f(n)=1^2+2^2+3^2+\cdots+n^2$에 대하여 $\sum_{k=1}^{20}\dfrac{2k+1}{f(k)}$의 값은?

① $\dfrac{40}{7}$ ② $\dfrac{45}{8}$ ③ $\dfrac{17}{3}$ ④ $\dfrac{57}{10}$ ⑤ $\dfrac{63}{11}$

해설

$f(n)=1^2+2^2+3^2+\cdots+n^2$

$=\sum_{k=1}^{20}k^2=\dfrac{n(n+1)(2n+1)}{6}$ 이므로

$\sum_{k=1}^{20}\dfrac{2k+1}{f(x)}=\sum_{k=1}^{20}\dfrac{2k+1}{\dfrac{k(k+1)(2k+1)}{6}}$

$=\sum_{k=1}^{20}\dfrac{6}{k(k+1)}=6\sum_{k=1}^{20}\left(\dfrac{1}{k}-\dfrac{1}{k+1}\right)$

$=6\left(1-\dfrac{1}{21}\right)=6\times\dfrac{20}{21}=\dfrac{40}{7}$

7. 수열의 합 $\sum_{k=1}^{n} \frac{2}{k(k+1)(k+2)}$ 의 값은?

① $\frac{n(n-3)}{(n+1)(n+2)}$
② $\frac{n(n+3)}{2(n+1)(n+2)}$
③ $\frac{n(n+6)}{3(n+1)(n+2)}$
④ $\frac{2n(n+3)}{(n+1)(n+3)}$
⑤ $\frac{n(n+1)}{4(n+1)(n+2)}$

해설

$\frac{2}{k(k+1)(k+2)} = \frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)}$ 이므로

(준식)$= \sum_{k=1}^{n} \left\{ \frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)} \right\}$

$$= \left(\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right) + \left(\frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} \right) + \cdots$$

$$+ \left\{ \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right\}$$

$$= \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+3)}{2(n+1)(n+2)}$$

8. $\dfrac{1}{3^2-1}+\dfrac{1}{5^2-1}+\dfrac{1}{7^2-1}+\cdots+\dfrac{1}{21^2-1}$ 의 값은?

① $\dfrac{1}{22}$ ② $\dfrac{3}{22}$ ③ $\dfrac{5}{22}$ ④ $\dfrac{7}{22}$ ⑤ $\dfrac{9}{22}$

해설

$$a_n=\frac{1}{(2n+1)^2-1}=\frac{1}{(2n+1-1)(2n+1+1)}$$

$$=\frac{1}{2n\cdot(2n+2)}$$

$$=\frac{1}{4n(n+1)}$$

$$=\frac{1}{4}\cdot\frac{1}{n+1-n}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$\sum_{k=1}^{10}a_k$

$$=\frac{1}{4}\cdot\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{10}-\frac{1}{11}\right)$$

$$=\frac{1}{4}\left(1-\frac{1}{11}\right)=\frac{10}{44}=\frac{5}{22}$$

9. $1+\dfrac{1}{1+2}+\dfrac{1}{1+2+3}+\cdots+\dfrac{1}{1+2+3+\cdots+n}$ 의 값을 구하면?

① $\dfrac{n}{n+1}$ ② $\dfrac{2n}{n+1}$ ③ $\dfrac{3n}{n+1}$ ④ $\dfrac{4n}{n+1}$ ⑤ $\dfrac{5n}{n+1}$

해설

(주어진 식)$=\sum_{k=1}^{n}\dfrac{1}{\dfrac{k(k+1)}{2}}$

$=2\sum_{k=1}^{n}\left(\dfrac{1}{k}-\dfrac{1}{k+1}\right)$

$=2\left(\dfrac{1}{1}-\dfrac{1}{2}+\dfrac{1}{2}-\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\dfrac{1}{n+1}\right)$

$=2\left(1-\dfrac{1}{n+1}\right)=\dfrac{2n}{n+1}$

10. $S=\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+\cdots+\dfrac{1}{19\cdot 20}$ 일 때, $100S$ 의 값은?

① 95 ② 100 ③ 105 ④ 110 ⑤ 115

해설

$$S=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{19\cdot 20}$$

$$=\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{19}-\frac{1}{20}$$

$$=1-\frac{1}{20}=\frac{19}{20}$$

$$\therefore 100S=100\times\frac{19}{20}=95$$

11. $\sum_{k=1}^{n} \dfrac{1}{(2k-1)(2k+1)}$ 의 값은?

① $\dfrac{1}{n+1}$　　② $\dfrac{2n}{n+1}$　　③ $\dfrac{n}{2n+1}$

④ $\dfrac{n}{n+2}$　　⑤ $\dfrac{2n}{2n+1}$

해설

(준 식)$= \dfrac{1}{2}\sum_{k=1}^{n}\left\{\dfrac{1}{2k-1} - \dfrac{1}{2k+1}\right\}$

$= \dfrac{1}{2}\cdot\left\{\left(1-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{5}\right)+\left(\dfrac{1}{5}-\dfrac{1}{7}\right)+\cdots\right\}$

$+ \dfrac{1}{2}\left\{\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right)\right\}$

$= \dfrac{1}{2}\left(1-\dfrac{1}{2n+1}\right) = \dfrac{n}{2n+1}$

12. $\sum_{k=1}^{49} \frac{1}{\sqrt{k}+\sqrt{k+1}} = a\sqrt{2}+b$일 때, $a+b$의 값은?

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

해설

$\sum_{k=1}^{49} \frac{1}{\sqrt{k}+\sqrt{k+1}}$

$= \sum_{k=1}^{49} \frac{\sqrt{k}-\sqrt{k+1}}{(\sqrt{k}+\sqrt{k+1})(\sqrt{k}-\sqrt{k+1})}$

$= \sum_{k=1}^{49}(\sqrt{k}-\sqrt{k+1})$

$= -\left\{(\sqrt{1}-\sqrt{2})+(\sqrt{2}-\sqrt{3})+\cdots\right\}$

$+\left\{(\sqrt{49}-\sqrt{50})\right\}$

$= -(1-\sqrt{50}) = 5\sqrt{2}-1$

따라서, $a=5,\ b=-1$에서 $a+b=4$

13. $\sum_{k=1}^{80}(\sqrt{k}-\sqrt{k+1})$ 의 값은?

① -5 ② -7 ③ -8 ④ -79 ⑤ -80

해설

$\sum_{k=1}^{80}(\sqrt{k}-\sqrt{k+1})$

$=\sqrt{1}-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\cdots+\sqrt{80}-\sqrt{81}$

$=\sqrt{1}-\sqrt{81}$

$=1-9=-8$

14. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$ 의 값은?

① $\frac{n(3n+5)}{4(n+1)(n+2)}$　② $\frac{n(3n+5)}{4(2n+1)(n+2)}$

③ $\frac{n(3n+5)}{(n+1)(n+2)}$　④ $\frac{n(3n+4)}{4(n+1)(n+2)}$

⑤ $\frac{n(3n+4)}{2(n+1)(n+2)}$

해설

$$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}$$

$$=\sum_{k=1}^{n}\frac{1}{k(k+2)}$$

$$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$$

$$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\}$$

$$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$$

$$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$=\frac{n(3n+5)}{4(n+1)(n+2)}$$

15. $\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\dfrac{1}{5\cdot 7}+\cdots+\dfrac{1}{(2n-1)(2n+1)}$ 의 값은?

① $\dfrac{n}{2n-1}$ ② $\dfrac{2n}{2n-1}$ ③ $\dfrac{n}{2n+1}$

④ $\dfrac{2n}{2n+1}$ ⑤ $\dfrac{n}{2n+3}$

해설

$\dfrac{1}{(2n-1)(2n+1)}=\dfrac{1}{2}\left(\dfrac{1}{(2k-1)(2k+1)}\right)$ 임을 이용한다.

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}$$

$$=\sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}$$

$$=\sum_{k=1}^{n}\frac{1}{2}\left(\frac{1}{(2k-1)(2k+1)}\right)$$

$$=\frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\}$$

$$+\cdots+\frac{1}{2}\left\{\left(\frac{1}{(2n-1)(2n+1)}\right)\right\}$$

$$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$$

$$=\frac{n}{2n+1}$$

16. $\sum_{k=1}^{n} \dfrac{1}{4k^2-1}$ 의 값은?

① $\dfrac{1}{n+1}$ ② $\dfrac{n}{n+1}$ ③ $\dfrac{2n}{n+1}$

④ $\dfrac{n}{2n+1}$ ⑤ $\dfrac{2n}{2n+3}$

해설

$$(\text{주어진 식}) = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)$$

$$\frac{1}{2}\left\{\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right)\right\}$$

$$+ \cdots + \frac{1}{2}\left\{\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)\right\}$$

$$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$$

17. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ 의 값은?

① $\frac{1}{6}$ ② $\frac{1}{3}$ ③ $\frac{1}{2}$ ④ $\frac{2}{3}$ ⑤ $\frac{5}{6}$

해설

(준 식)$=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$

$=1-\frac{1}{6}=\frac{5}{6}$

18. $\sum_{k=1}^{n} \dfrac{1}{k^2+k}$ 의 값은?

① $\dfrac{1}{n+1}$ ② $\dfrac{n}{n+1}$ ③ $\dfrac{2n}{n+1}$

④ $\dfrac{2n}{2n+1}$ ⑤ $\dfrac{2n}{2n+3}$

해설

(주어진 식)$= \sum_{k=1}^{n} \left(\dfrac{1}{k} - \dfrac{1}{k+1}\right)$

$= \left(\dfrac{1}{1} - \dfrac{1}{2}\right) + \left(\dfrac{1}{2} - \dfrac{1}{3}\right) + \left(\dfrac{1}{3} - \dfrac{1}{4}\right) + \cdots + \left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

$= 1 - \dfrac{1}{n+1} = \dfrac{n}{n+1}$

19. 수열 $\frac{1}{2^2-1}, \frac{1}{3^2-1}, \frac{1}{4^2-1}, \frac{1}{5^2-1}, \cdots$ 의 첫째항부터 제 n항까지의 합을 구하면?

① $\frac{n+2}{2(n+1)}$

② $\frac{2n}{(n+1)(n+2)}$

③ $\frac{n(3n+5)}{4(n+1)(n+2)}$

④ $\frac{2n+5}{2(n+3)}$

⑤ $\frac{2n(n+1)}{(n+3)(n+5)}$

해설

$$a_k = \frac{1}{(k+1)^2-1} = \frac{1}{k(k+2)}$$

$= \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right)$ 이므로

$$S_n = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+2}\right)$$

$$= \frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\} + \cdots +$$

$$\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$$

$$= \frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$= \frac{n(3n+5)}{4(n+1)(n+2)}$$

20. $\sum_{k=1}^{n} a_k = n^2 + 3n$ 일 때, $\sum_{k=1}^{10} \frac{1}{a_k a_{k+1}}$ 의 값은?

① $\frac{1}{24}$ ② $\frac{1}{48}$ ③ $\frac{5}{16}$ ④ $\frac{5}{24}$ ⑤ $\frac{5}{48}$

해설

$a_n = S_n - S_{n-1}$

$= n^2 + 3n - \{(n-1)^2 + 3(n-1)\} = 2n + 2(n \geq 2)$

$a_1 = 1 + 3 = 2 + 2 = 4$ 이므로, $a_n = 2n + 2(n \geq 1)$

$$\sum_{k=1}^{10} \frac{1}{a_k a_{k+1}} = \sum_{k=1}^{10} \frac{1}{(2k+2)(2k+4)}$$

$$= \frac{1}{4} \sum_{k=1}^{10} \left(\frac{1}{k+1} - \frac{1}{k+2} \right)$$

$$= \frac{1}{4} \left\{ \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{11} - \frac{1}{12} \right) \right\}$$

$$= \frac{1}{4} \left(\frac{1}{2} - \frac{1}{12} \right) = \frac{5}{24}$$

21. $\sum_{k=1}^{n} \dfrac{1}{(2k-1)(2k+1)}$ 의 값은?

① $\dfrac{1}{n+1}$ ② $\dfrac{2n}{n+1}$ ③ $\dfrac{n}{2n+1}$

④ $\dfrac{n}{n+2}$ ⑤ $\dfrac{2n}{2n+1}$

해설

준 식) $= \dfrac{1}{2}\sum_{k=1}^{n}\left\{\dfrac{1}{2k-1} - \dfrac{1}{2k+1}\right\}$

$= \dfrac{1}{2} \cdot \left\{\left(1-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{5}\right)+\left(\dfrac{1}{5}-\dfrac{1}{7}\right)\right\} + \cdots +$

$\dfrac{1}{2}\left\{\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right)\right\}$

$= \dfrac{1}{2}\left(1-\dfrac{1}{2n+1}\right)$

$= \dfrac{n}{2n+1}$

22. $\sum_{k=1}^{200} \frac{1}{k(k+1)}$ 의 값은?

① $\frac{101}{100}$ ② $\frac{100}{101}$ ③ $\frac{200}{201}$ ④ $\frac{110}{101}$ ⑤ $\frac{201}{200}$

해설

$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ 이므로

$\sum_{k=1}^{200} \frac{1}{k(k+1)} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots +$
$\left(\frac{1}{199} - \frac{1}{200}\right) + \left(\frac{1}{200} - \frac{1}{201}\right)$
$= \frac{1}{1} - \frac{1}{201} = \frac{200}{201}$