1. $\sum_{k=1}^{5} a_k = 20$, $\sum_{k=1}^{5} b_k = 5$일 때, $\sum_{k=1}^{5}(2a_k - b_k - 1)$의 값은?

① 15 ② 20 ③ 25 ④ 30 ⑤ 35

해설

(주어진 식)

$= 2\sum_{k=1}^{5} a_k - \sum_{k=1}^{5} b_k - \sum_{k=1}^{5} 1$

$= 2 \cdot 20 - 5 - 5$

$= 30$

2. $\sum_{k=1}^{100} a_k = 10$, $\sum_{k=1}^{100} a_k^2 = 20$, 일 때, $\sum_{k=1}^{100}(a_k+1)^2+\sum_{k=1}^{100}(a_k-2)^2$ 의 값은?

① 520 ② 540 ③ 560 ④ 580 ⑤ 600

해설

$$\sum_{k=1}^{100}(a_k+1)^2+\sum_{k=1}^{100}(a_k-2)^2$$
$$=\sum_{k=1}^{100}(2a_k^2-2a_k+5)$$
$$=2\cdot\sum_{k=1}^{100} a_k^2-2\cdot\sum_{k=1}^{100} a_k+\sum_{k=1}^{100} 5$$
$$=2\cdot 20-2\cdot 10+500$$
$$=40-20+500=520$$

3. $\sum_{k=1}^{5} a_k = 5$, $\sum_{k=1}^{5} b_k = 7$일 때, $\sum_{k=1}^{5}(3a_k + 2b_k)$의 값은?

① 21　② 22　③ 23　④ 24　⑤ 29

해설

$\sum_{k=1}^{5}(3a_k + 2b_k) = \sum_{k=1}^{5} 3a_k + \sum_{k=1}^{5} 2b_k$

$= 3\sum_{k=1}^{5} a_k + 2\sum_{k=1}^{5} b_k$

$= 3 \times 5 + 2 \times 7 = 15 + 14 = 29$

4. $\sum_{k=1}^{10} k^3$ 의 값을 구하여라.

▶ **답 :**

▷ **정답 :** 3025

해설

$$\sum_{k=1}^{10} k^3 = \frac{10 \cdot 11}{2} \cdot \frac{10 \cdot 11}{2} = 3025$$

5. 두 수열 $\{a_n\}$, $\{b_n\}$에 대하여 $a_n = \frac{n}{3}$, $b_n = 2^n$일 때, $\sum_{k=1}^{5}(a_k + b_k)$의 값은?

① 61　② 63　③ 65　④ 67　⑤ 69

해설

$$\sum_{k=1}^{5}(a_k + b_k) = \sum_{k=1}^{5} a_k + \sum_{k=1}^{5} b_k = \sum_{k=1}^{5} \frac{k}{3} + \sum_{k=1}^{5} 2^k$$

$$= \frac{1}{3} \cdot \frac{5 \cdot 6}{2} + \frac{2(2^5 - 1)}{2 - 1} = 67$$

6. 다음 식의 값은?

$$\sum_{k=1}^{10}(k^2+k)-\sum_{k=4}^{10}(k^2+k)$$

① 14　　② 16　　③ 18　　④ 20　　⑤ 22

해설

(준 식)$=\sum_{k=1}^{3}(k^2+k)=(1^2+1)+(2^2+2)+(3^2+3)=20$

7. $\sum_{k=1}^{10} a_k = 3$, $\sum_{k=1}^{10} b_k = 5$ 일 때, $\sum_{k=1}^{10}(a_k + 2b_k - 1)$ 의 값은?

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

해설

$$\sum_{k=1}^{10}(a_k + 2b_k - 1) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} 2b_k - \sum_{k=1}^{10} 1$$
$$= \sum_{k=1}^{10} a_k + 2\sum_{k=1}^{10} b_k - \sum_{k=1}^{10} 1$$
$$= 3 + 2 \times 5 - 10 = 3$$

8. $\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 470

해설

$$\sum_{k=11}^{15} k^2 - \sum_{k=1}^{10} k^2 = \left(\sum_{k=1}^{15} k^2 - \sum_{k=1}^{10} k^2\right) - \sum_{k=1}^{10} k^2$$
$$= \sum_{k=1}^{15} k^2 - 2\sum_{k=1}^{10} k^2$$
$$= \frac{15 \cdot 16 \cdot 31}{6} - 2 \cdot \frac{10 \cdot 11 \cdot 21}{6} = 470$$

9. $\sum_{k=1}^{n} a_k = \mathrm{A}$, $\sum_{k=1}^{n} b_k = \mathrm{B}$일 때, 다음 중 옳지 않은 것은?

① $\sum_{k=1}^{n}(a_k + b_k) = \mathrm{A} + \mathrm{B}$

② $\sum_{k=1}^{n}(a_k - b_k) = \mathrm{A} - \mathrm{B}$

③ $\sum_{k=1}^{n} ca_k = c\mathrm{A}$(단, c는 상수)

④ $\sum_{k=2}^{n+1} b_{k-1} = \mathrm{B} - 1$

⑤ $\sum_{k=1}^{n}(a_k + c) = \mathrm{A} + cn$(단, c는 상수)

해설

$\sum_{k=2}^{n+1} b_{k-1} = \sum_{k=1}^{n} b_k = \mathrm{B}$

따라서 , ④가 옳지 않다.

10. 수열 $\{a_n\}$이 $a_1 = 1,\ a_{10} = 30$을 만족할 때 $\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$의 값은?

① 26　② 27　③ 28　④ 29　⑤ 30

해설

$\sum_{k=1}^{9} a_{k+1} - \sum_{k=2}^{10} a_{k-1}$
$= (a_2 + a_3 + \cdots + a_9 + a_{10}) -$
$(a_1 + a_2 + \cdots + a_9)$
$= -a_1 + a_{10} = -1 + 30 = 29$

11. $\sum_{k=3}^{10} k(k+2)$ 의 값은?

① 460　　② 468　　③ 478　　④ 480　　⑤ 484

해설

$\sum_{k=1}^{10} k(k+2) = \sum_{k=1}^{10} k(k+2) - \sum_{k=1}^{2} k(k+2)$

$= \sum_{k=1}^{10}(k^2+2k) - \sum_{k=1}^{2}(k^2+2k)$

$= \sum_{k=1}^{10} k^2 + 2\sum_{k=1}^{10} k - (3+8)$

$$= \frac{10 \cdot 11 \cdot 21}{6} + 2 \cdot \frac{10 \cdot 11}{2} - 11$$

$= 385 + 110 - 11$

$= 484$

12. $\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1)=62$를 만족하는 자연수 n의 값을 구하여라.

▶ **답 :**

▷ **정답 :** 7

해설

$\sum_{k=1}^{n}(k^2+1)-\sum_{k=1}^{n-1}(k^2-1)$

$=\sum_{k=1}^{n}(k^2+1)-\left\{\sum_{k=1}^{n}(k^2-1)-(n^2-1)\right\}$

$=\sum_{k=1}^{n}\left\{(k^2+1)-(k^2-1)\right\}+(n^2-1)$

$=\sum_{k=1}^{n}2+(n^2-1)=n^2+2n-1=62$

이것을 정리하여 인수분해하면 $(n+9)(n-7)=0$

따라서 $n=-9$ 또는 $n=7$

그런데 $n>0$이므로 $n=7$

13. $\sum_{k=2}^{n+1}(k^2+k+1)-\sum_{k=1}^{n-1}(k^2-k-1)$ 을 n 에 관한 식으로 나타낸 것은?

① n^2+5n-1 ② $3n^2+5n-1$ ③ $4n^2+2n-1$

④ $4n^2+5n-1$ ⑤ $5n^2+5n-1$

해설

$\sum_{k=2}^{n+1}(k^2+k+1)-\sum_{k=2}^{n-1}(k^2-k-1)$

$=\sum_{k=2}^{n}(k^2+k+1)+\left\{(n+1)^2+(n+1)+1\right\}-3-\left\{\sum_{k=1}^{n}(k^2-k-1)-(n^2-n-1)\right\}$

$=\sum_{k=1}^{n}\left\{(k^2+k+1)-(k^2-k-1)\right\}+n^2+2n+1+n+2-3+n^2-n-1$

$=\sum_{k=1}^{n}(2k+2)+2n^2+2n-1$

$$=2\cdot\frac{n(n+1)}{2}+2n+2n^2+2n-1$$

$=3n^2+5n-1$

14. 수열 $\{a_n\}$에 대하여 $\sum_{k=1}^{n}(a_{2k-1}+a_{2k})=8n^2+10n$일 때, $\sum_{k=1}^{10}a_k$의 값을 구하여라.

▶ 답 :

▷ 정답 : 250

해설

$\sum_{k=1}^{10}a_k=a_1+a_2+a_3+\cdot+a_{10}$
$=(a_1+a_2)+(a_3+a_4)+\cdots+(a_9+a_{10})$
$=\sum_{k=1}^{5}(a_{2k-1}+a_{2k})$
$=8\times5^2+10\times5=250$

15. 다음 등식이 성립하도록 하는 c의 값을 구하여라.

$$\sum_{k=11}^{100}(k-2)^2=\sum_{k=11}^{100}k^2-4\sum_{k=11}^{100}k+c$$

▶ 답 :

▷ 정답 : 360

해설

$$\sum_{k=11}^{100}(k-2)^2=\sum_{k=11}^{100}(k^2-4k+4)$$
$$=\sum_{k=11}^{100}-4\sum_{k=11}^{100}k+\sum_{k=11}^{100}4$$
$$\therefore\ c=\sum_{k=11}^{100}4=4+4+\cdots+4=4\times 90=360$$

16. $S=\sum_{k=1}^{10}k+\sum_{k=2}^{10}k+\sum_{k=3}^{10}k+\cdots+\sum_{k=9}^{10}k+\sum_{k=10}^{10}k$일 때, $\frac{1}{5}S$의 값을 구하여라.

▶ 답 :

▷ 정답 : 77

해설

$S=\sum_{k=1}^{10}k+\sum_{k=2}^{10}k+\sum_{k=3}^{10}k+\cdots+\sum_{k=9}^{10}k+\sum_{k=10}^{10}k$

$=1+2+3+4+\cdots+10$

$+2+3+4+\cdots+10$

$3+4+\cdots+10$

$\vdots$

$+10$

$=1+2^2+3^2+4^2+\cdots+10^2$

$=\dfrac{10\times11\times21}{6}=385$

$\therefore\ \frac{1}{5}S=77$

17. $\sum_{k=1}^{n} a_k = n^2 + 2n$ 일 때, $\sum_{k=1}^{3}(a_k + 1)^2 - \sum_{k=1}^{3}(a_k - 1)^2$ 의 값을 구하여라.

▶ 답 :

▷ 정답 : 60

해설

$\sum_{k=1}^{3}(a_k + 1)^2 - \sum_{k=1}^{3}(a_k - 1)^2$
$= \sum_{k=1}^{3}(a_k + 2a_k + 1) - \sum_{k=1}^{3}(a_k^2 - 2a_k + 1)$
$= 4\sum_{k=1}^{3} a_k = 4(3^2 + 2 \times 3) = 60$

18. $(2^2+1)+(3^2+3)+(4^2+5)+\cdots+(10^2+17)$ 의 값은?

① 465 ② 466 ③ 467 ④ 468 ⑤ 469

해설

$\sum_{k=1}^{9}\left\{(k+1)^2+(2k-1)\right\}=\sum_{k=1}^{9}(k^2+4k)$

$=\sum_{k=1}^{9}k^2+\sum_{k=1}^{9}4k$

$=\dfrac{9\cdot 10\cdot 19}{6}+4\cdot\dfrac{9\cdot 10}{2}=285+180=465$

19. $\sum_{k=1}^{10}(a_k+1)^2=60$, $\sum_{k=1}^{10}(a_k-1)^2=20$일 때, $\sum_{k=1}^{10}a_k$의 값은?

① 10 ② 20 ③ 30 ④ 40 ⑤ 50

해설

$\sum_{k=1}^{10}(a_k+1)^2=60$에서 $\sum_{k=1}^{10}(a_k^2+2a_k+1)=60$

$\sum_{k=1}^{10}a_k^2+2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1=60\cdots\cdots$ ㉠

$\sum_{k=1}^{10}(a_k-1)^2=20$에서 $\sum_{k=1}^{10}(a_k^2-2a_k+1)=20$

$\sum_{k=1}^{10}a_k^2-2\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}1=20\cdots\cdots$ ㉡

㉠- ㉡을 계산하면

$4\sum_{k=1}^{10}a_k=40 \quad \therefore \sum_{k=1}^{10}a_k=10$

20. $\sum_{k=1}^{n} = n^2 + 1$일 때, 다음 보기에서 옳은 것을 모두 고른 것은?

보기

ㄱ $a_5 = 9$

ㄴ $\sum_{k=1}^{n} a_{2k} = 2n^2 + n$

ㄷ $\sum_{k=1}^{n} a_{2k-1} = 2n^2 - n + 1$

① ㄱ ② ㄴ ③ ㄱ, ㄴ

④ ㄴ, ㄷ ⑤ ㄱ, ㄴ, ㄷ

해설

ㄱ $\sum_{k=1}^{n} a_k = S_n$이라 하면

$a_5 = S_5 - S_4 = (5^2+1) - (4^2+1) = 9$(참)

ㄴ $S_n = n^2 + 1$이므로

$a_n = S_n - S_{n-1} = n^2 + 1 - \{(n-1)^2 + 1\}$

$= 2n - 1 (n \geq 2)$

$\therefore\ a_{2n} = 2(2n) - 1 = 4n - 1 (n \geq 1)$

$\therefore\ \sum_{k=1}^{n} a_{2k} = \sum_{k=1}^{n}(4k-1) = 4\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = 2n^2 + n$(참)

ㄷ $a_n = 2n - 1 (n \geq 2)$이고 $a_1 = S_1 = 2$이다. $\sum_{k=1}^{n} a_{2k-1} =$ $2 + \sum_{k=2}^{n}\{2(2k-1) - 1\} = 2 + \sum_{k=2}^{n}(4k-3)$

$= 2 + \sum_{k=1}^{n}(4k-3) - 1 = 2 + 4\sum_{k=1}^{n} k - \sum_{k=1}^{n} 3 - 1$

$= 1 + 4 \cdot \dfrac{n(n+1)}{2} - 3n = 2n^2 - n + 1$(참)

따라서, 보기 중에서 옳은 것은 ㄱ, ㄴ, ㄷ이다.