1. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ 의 값은?

① $\frac{1}{6}$ ② $\frac{1}{3}$ ③ $\frac{1}{2}$ ④ $\frac{2}{3}$ ⑤ $\frac{5}{6}$

해설

(준 식)$=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{3}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}$

$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$

$=1-\frac{1}{6}=\frac{5}{6}$

2. $\sum_{k=1}^{50} \sqrt{(2k+1) - 2\sqrt{k(k+1)}}$ 의 값을 α라 할 때, 자연수 n에 대하여 $n < \alpha < n+1$이 성립한다. 이때 n의 값은?

① 5　　② 6　　③ 7　　④ 8　　⑤ 9

해설

$\sum_{k=1}^{50} \sqrt{(2k+1) - 2\sqrt{k(k+1)}}$

$= \sum_{k=1}^{50} (\sqrt{k+1} - \sqrt{k})$

$= (\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + \cdots + (\sqrt{51} - \sqrt{50})$

$= \sqrt{51} - 1$

$7 < \sqrt{51} < 8$이므로 $6 < \sqrt{51} - 1 < 7$

$\therefore n = 6$

3. 수열 $\frac{1}{2^2-1}, \frac{1}{3^2-1}, \frac{1}{4^2-1}, \frac{1}{5^2-1}, \cdots$ 의 첫째항부터 제 n항까지의 합을 구하면?

① $\frac{n+2}{2(n+1)}$ ② $\frac{2n}{(n+1)(n+2)}$

③ $\frac{n(3n+5)}{4(n+1)(n+2)}$ ④ $\frac{2n+5}{2(n+3)}$

⑤ $\frac{2n(n+1)}{(n+3)(n+5)}$

해설

$a_k = \frac{1}{(k+1)^2-1} = \frac{1}{k(k+2)}$

$= \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right)$ 이므로

$S_n = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+2}\right)$

$= \frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)\right\} + \cdots +$

$\frac{1}{2}\left\{\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right\}$

$= \frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$

$= \frac{n(3n+5)}{4(n+1)(n+2)}$

4. 함수 $f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$ 에 대하여 $\sum_{k=1}^{20} \dfrac{2k+1}{f(k)}$ 의 값은?

① $\dfrac{40}{7}$ ② $\dfrac{45}{8}$ ③ $\dfrac{17}{3}$ ④ $\dfrac{57}{10}$ ⑤ $\dfrac{63}{11}$

해설

$f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$= \sum_{k=1}^{20} k^2 = \dfrac{n(n+1)(2n+1)}{6}$ 이므로

$\sum_{k=1}^{20} \dfrac{2k+1}{f(x)} = \sum_{k=1}^{20} \dfrac{2k+1}{\dfrac{k(k+1)(2k+1)}{6}}$

$= \sum_{k=1}^{20} \dfrac{6}{k(k+1)} = 6\sum_{k=1}^{20} \left(\dfrac{1}{k} - \dfrac{1}{k+1}\right)$

$= 6\left(1 - \dfrac{1}{21}\right) = 6 \times \dfrac{20}{21} = \dfrac{40}{7}$

5. 자연수 n에 대하여 $\sqrt{n}$의 정수 부분을 $f(n)$이라 하자. 예를 들면, $f(5)=2$이다. 이때, $\sum_{n=1}^{120} \dfrac{1}{2f(n)+1}$의 값은?

① 10　② 12　③ 20　④ 24　⑤ 36

해설

$f(1)=1,\ f(2)=f(3)=1,$
$f(4)=f(2^2)=2,\ f(5)=f(6)=f(7)=f(8)=2,$
$f(9)=f(3^2)=3,\ f(10)=f(11)=\cdots=f(15)=3,$
$\vdots$
$f(120)=10,\ f(121)=f(11^2)=11$이므로
$\sum_{n=1}^{120} \dfrac{1}{2f(n)+1} = \dfrac{1}{2\cdot 1+1}+\dfrac{1}{2\cdot 1+1}+\dfrac{1}{2\cdot 1+1}+\dfrac{1}{2\cdot 2+1}+$
$\cdots+\dfrac{1}{2\cdot 10+1}$
$=\dfrac{1}{3}\cdot 3+\dfrac{1}{5}\cdot 5+\dfrac{1}{7}\cdot 7+\cdots+\dfrac{1}{21}\cdot 21$
$=1+1+1+1+\cdots+1=1\cdot 10=10$

6. 다음 수열의 합을 구하여라.

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9$$

▶ 답 :

▷ 정답 : 8194

해설

$S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9 \cdots$ ㉠

$2S = 1 \cdot 2^2 + 2 \cdot 2^3 + \cdots + 8 \cdot 2^9 + 9 \cdot 2^{10} \cdots$ ㉡

이므로 ㉠-㉡을 하면

$$\begin{aligned} -S &= \frac{2(2^9 - 1)}{2 - 1} - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 2 - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 18 \cdot 2^9 - 2 \\ &= -16 \cdot 2^9 - 2 \end{aligned}$$

$\therefore S = 2^{13} + 2 = 1024 \times 8 + 2 = 8194$