1. $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ 의 값은?

① $\frac{1}{6}$ ② $\frac{1}{3}$ ③ $\frac{1}{2}$ ④ $\frac{2}{3}$ ⑤ $\frac{5}{6}$

해설

$$(\text{준 식}) = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{3}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$$

$$=1-\frac{1}{6}=\frac{5}{6}$$

2. $\sum_{k=1}^{50} \sqrt{(2k+1)-2\sqrt{k(k+1)}}$의 값을 α라 할 때, 자연수 n에 대하여 $n < \alpha < n+1$이 성립한다. 이때 n의 값은?

① 5 ② 6 ③ 7 ④ 8 ⑤ 9

해설

$$\sum_{k=1}^{50} \sqrt{(2k+1)-2\sqrt{k(k+1)}}$$

$$= \sum_{k=1}^{50} (\sqrt{k+1} - \sqrt{k})$$

$$= (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + \cdots + (\sqrt{51}-\sqrt{50})$$

$$= \sqrt{51} - 1$$

$7 < \sqrt{51} < 8$이므로 $6 < \sqrt{51} - 1 < 7$

$\therefore n = 6$

3. 수열 $\frac{1}{2^2-1}, \frac{1}{3^2-1}, \frac{1}{4^2-1}, \frac{1}{5^2-1}, \cdots$ 의 첫째항부터 제 n 항까지의 합을 구하면?

① $\frac{n+2}{2(n+1)}$
② $\frac{2n}{(n+1)(n+2)}$
③ $\frac{n(3n+5)}{4(n+1)(n+2)}$
④ $\frac{2n+5}{2(n+3)}$
⑤ $\frac{2n(n+1)}{(n+3)(n+5)}$

해설

$$a_k = \frac{1}{(k+1)^2-1} = \frac{1}{k(k+2)}$$

$$= \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right) \text{이므로}$$

$$S_n = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+2}\right)$$

$$= \frac{1}{2}\left\{\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right)\right\} + \cdots +$$

$$\frac{1}{2}\left\{\left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right)\right\}$$

$$= \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{n(3n+5)}{4(n+1)(n+2)}$$

4. 함수 $f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$ 에 대하여 $\sum_{k=1}^{20} \frac{2k+1}{f(k)}$ 의 값은?

① $\frac{40}{7}$ ② $\frac{45}{8}$ ③ $\frac{17}{3}$ ④ $\frac{57}{10}$ ⑤ $\frac{63}{11}$

해설

$f(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$= \sum_{k=1}^{20} k^2 = \frac{n(n+1)(2n+1)}{6}$ 이므로

$$\sum_{k=1}^{20} \frac{2k+1}{f(x)} = \sum_{k=1}^{20} \frac{2k+1}{\frac{k(k+1)(2k+1)}{6}}$$

$$= \sum_{k=1}^{20} \frac{6}{k(k+1)} = 6\sum_{k=1}^{20} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= 6\left(1 - \frac{1}{21}\right) = 6 \times \frac{20}{21} = \frac{40}{7}$$

5. 자연수 n에 대하여 $\sqrt{n}$의 정수 부분을 $f(n)$이라 하자. 예를 들면, $f(5)=2$이다. 이때, $\sum_{n=1}^{120}\frac{1}{2f(n)+1}$의 값은?

① 10 ② 12 ③ 20 ④ 24 ⑤ 36

해설

$f(1)=1,\ f(2)=f(3)=1,$
$f(4)=f(2^2)=2,\ f(5)=f(6)=f(7)=f(8)=2,$
$f(9)=f(3^2)=3,\ f(10)=f(11)=\cdots=f(15)=3,$

$\vdots$

$f(120)=10,\ f(121)=f(11^2)=11$이므로

$$\sum_{n=1}^{120}\frac{1}{2f(n)+1}=\frac{1}{2\cdot1+1}+\frac{1}{2\cdot1+1}+\frac{1}{2\cdot1+1}+\frac{1}{2\cdot2+1}+\cdots+\frac{1}{2\cdot10+1}$$

$$=\frac{1}{3}\cdot3+\frac{1}{5}\cdot5+\frac{1}{7}\cdot7+\cdots+\frac{1}{21}\cdot21$$

$$=1+1+1+1+\cdots+1=1\cdot10=10$$

6. 다음 수열의 합을 구하여라.

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9$$

▶ 답 :

▷ 정답 : 8194

해설

$S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 9 \cdot 2^9 \cdots$ ㉠

$2S = 1 \cdot 2^2 + 2 \cdot 2^3 + \cdots + 8 \cdot 2^9 + 9 \cdot 2^{10} \cdots$ ㉡

이므로 ㉠-㉡을 하면

$$\begin{aligned} -S &= \frac{2(2^9 - 1)}{2 - 1} - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 2 - 9 \cdot 2^{10} \\ &= 2 \cdot 2^9 - 18 \cdot 2^9 - 2 \\ &= -16 \cdot 2^9 - 2 \end{aligned}$$

$\therefore S = 2^{13} + 2 = 1024 \times 8 + 2 = 8194$